Multilateral Risk-Sharing with Manipulation*

Yair Antler

Tel Aviv University

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Abstract

We study multilateral risk-sharing when the state of nature is unverifiable, such that contracts are conditioned on a state-dependent signal (e.g., net earnings in a financial report). A subset of the agents can manipulate the signal's realization at some cost (e.g., by performing financial acrobatics) and as a result Pareto-optimal risk-sharing is precluded. The agents are able to write additional bilateral side-contracts without withdrawing from the prevailing risk-sharing agreements. Such side-contracts can be used to incentivize one of the parties to manipulate the signal. Using a weak stability notion we show that, in general, stable contracts are not constrained-Pareto-optimal. We derive closed-form solutions for the maximal possible coverage in a few settings (reinsurance of a local shock, joint venture) and show that it is significantly below the constrained-Pareto-optimal level of insurance. Moreover, it is non-monotone in the number of agents who can manipulate the signal.

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Correspondence: Yair Antler, The Eitan Berglas School of Economics, Tel Aviv University. *E-mail:* yair.an@gmail.com.

1 Introduction

It is well known that when risk-averse agents have access to Arrow-Debreu securities, they can share risk efficiently. In practice, however, state-contingent contracts are not always feasible, for a variety of reasons: the state of nature may be unobservable, unverifiable, or hard to assess to the point where state-contingent contracts are unenforceable or too costly to implement.

For these reasons, many risk-sharing contracts are contingent on verifiable variables that are informative about the state of nature, rather than contingent on the state of nature itself. For example, insurance contracts are often contingent on an appraisal rather than on actual damage. Other prominent examples are managerial compensation contracts, which are often contingent on the firm's net earnings as they appear in its financial reports rather than contingent on the firm's actual performance; derivatives, which are often contingent on the value of a stock rather than on its fundamentals; and benchmarks such as the inter-bank offered rates (a measure of the interest rate at which large banks can borrow from one another on an unsecured basis), which are often used in contracts to transfer risk related to fluctuations in general market-wide interest rates (Duffie and Stein, 2015). In the present paper, we focus on contracts of this kind and refer to the contractible variable as a *signal* about the state of nature.

Sharing risk in these types of contracts gives rise to a *moral hazard* problem that results from the agents' ability to *manipulate* the signal's realization by taking costly actions. Such costly actions include: forging an appraisal, or misreporting the occurrence of an insurable event; deferring recognition of some expenditure in order to change a firm's net earnings/inventory at a specific date; bailing out a distressed debtor when a contract is contingent on a third party's debt solvency; manipulating future prices in a commodity market by submitting large buy or sell orders on the spot market; manipulating the rate of a currency by providing the customers of a large investment bank with a false recommendation to buy/sell it; and hiring lobbyists to influence a regulator's action (on which a contract depends).

We study multilateral risk-sharing in an economy that consists of $n > 2$ agents. Risk is shared by using contracts that are contingent on a signal that reveals the state of nature. A risk-sharing contract sets transfers between the contracting agents contingent on the signal. Some of the agents have the ability to unilaterally manipulate the signal's realization by incurring some cost.

The timeline in the model is as follows. In period 0, contracts are signed by the agents. We often refer to the collection of these contracts as the *multilateral contract*.

At the beginning of period 1, the state of nature and its corresponding signal are realized. When for each agent the cost of manipulating the signal is greater than the corresponding benefit, the multilateral contract is said to be incentive compatible, and we assume that the signal *perfectly reveals* the state of nature (i.e., there is no manipulation). If the multilateral contract is not incentive compatible, then the agent who sets the value of the signal to his preferred realization is the one who benefits the most from doing so.[1] In period 2, the contracts are enforced according to the signal that was set in period 1.

To illustrate some of the model's features, we present the following example.

Example 1 *There are two states, good (g) and bad (b). Kate and Bruce are exposed to a negative shock of* 100 *dollars each in state* b*. There are* $n-2$ *risk-neutral insurers. Risk-sharing contracts are contingent on an appraisal* $y \in \{b, g\}$ *made by a certified appraiser. Kate and Bruce both know the appraiser. In state* g*, each of them can pay the appraiser a bribe of* 90 *dollars so that he will change the appraisal to* b*.*

Observe that full insurance is not incentive compatible since it incentivizes Kate or Bruce to bribe the appraiser in state g*. Because of the moral hazard, each of them can receive a coverage of at most* 90 *dollars. Since the insurers are risk-neutral, this is also the coverage that Kate and Bruce receive in every multilateral contract that is individually rational and not Pareto-dominated. We refer to such contracts as constrained-efficient contracts.*

The main novelty in this work is that, in the contracting stage, agents can add new bilateral side-contracts to the prevailing multilateral contract without withdrawing from it. The purpose of an additional side-contract can be either legitimate mutual insurance or to incentivize one of the contracting counter-parties to manipulate the signal. This introduces a new source of instability into multilateral risk-sharing since these contracts impose an externality on third parties.

We use Example 1 to demonstrate how a pair of agents may benefit from the addition of a side-contract that incentivizes one of them to manipulate the signal. Let us consider a multilateral risk-sharing contract in which Bruce and Kate receive coverage of 90 dollars each. Recall that this is the maximal coverage that each of them can receive without violating his/her incentive constraint. Both Kate and Bruce are better off if, in the contracting stage, they add a side-contract in which Bruce pays Kate a small $\varepsilon > 0$ if and only if $y = b$. This contract violates Kate's incentive constraint and

[1]In Appendix A, we discuss this assumption and show that it is a result of an equilibrium in two natural manipulation games.

incentivizes her to bribe the appraiser in state g. Bruce is better off since he guarantees his preferred appraisal by paying a small cost of ε. Roughly speaking, the bilateral contract between Kate and Bruce violates the stability of the multilateral risk-sharing contract because it makes both of them better off when the possibility of ex-post manipulation is taken into account. Note that Kate and Bruce's side-contract imposes a negative externality on insurers who provide them with coverage. An insurer who predicts this externality would be unwilling to provide Kate or Bruce with coverage.

Our main research objective is to study whether and to what extent the combination of the agents' ability to write bilateral side-contracts and the manipulability of the signal restricts multilateral risk-sharing. The answer to this question has potential implications on numerous risk-sharing environments that are exposed to manipulation. In particular, it has implications on the level of risk that can be transferred using contracts that are conditioned on economic benchmarks such as the inter-bank offered rates and on the manipulability of these benchmarks. These benchmarks have an important role in reducing market frictions (Duffie and Stein, 2015). In designing such benchmarks, manipulability is a major consideration (see, e.g., Duffie and Stein, 2015 and Duffie and Dworczak, 2015).

We study the multilateral risk-sharing problem by using a stability notion that is inspired by Jackson and Wolinsky's (1996) pairwise stability. Since writing a bilateral contract requires mutual consent and this notion is easily applicable, we find it to be a good starting point for our analysis. A multilateral contract is *pairwise stable* if there is no pair of agents who are better off adding a new bilateral side-contract (without withdrawing from the prevailing multilateral contract). It turns out that a pairwise stable multilateral contract does not exist. When the agents have low insurance coverage, there is a pair of agents who are better off adding a side-contract that increases their coverage. When the agents' insurance coverage is high, there are two agents who are better off adding a new bilateral side-contract that incentivizes one of them to manipulate the signal.

Pairwise stability involves the following restrictive implicit assumption: each agent i who takes part in some deviation believes that there are no additional deviations that make him worse off if he agrees to take part in this deviation. In particular, he believes that the counter-party to the deviation does not have an ulterior motive such as other side-contracts (with other agents) that are not observed by i. This assumption follows from the fact that pairwise stability considers the addition of one side-contract at a time. This assumption is particularly restrictive in the present paper's setting since the attractiveness of a contract (and, in particular, of taking part in a deviation) is affected by the actions of third parties.

We relax the above-mentioned assumption by developing a new weaker stability notion, which we shall refer to as *weak stability*. Weak stability incorporates considerations that are familiar from the Nash equilibrium refinements literature into a notion of stability in the tradition of cooperative game theory. The idea behind weak stability is that each pairwise deviation can be viewed as if it were initiated by one of the deviating counter-parties, say, i. An agent j who receives an offer to take part in this deviation conjectures which other deviations agent i has initiated with other agents because they have an effect on the attractiveness of i's offer. The only restriction we impose on agent j's conjecture is that it must rationalize the observed offer. That is, i's offer to j is required to make i better off, according to j's conjecture. We refer to such a conjecture as *consistent*. Agent j rejects i's offer if there exist consistent conjectured deviations that make j worse off if he agrees to i's offer.

Let us demonstrate this logic using Example 1. Consider a multilateral contract in which Kate receives a coverage of 90 dollars (i.e., she is indifferent whether to bribe the appraiser or not) and Bruce is not covered. Suppose Bruce offers one of the insurers, Susie, to write a bilateral side-contract in which she provides him with coverage in return for some premium. How should Susie respond to Bruce's offer? It depends on her beliefs about other deviations from the prevailing multilateral contract initiated by Bruce. Suppose Susie suspects that Bruce contemporaneously offered Kate a contract in which he pays her a small $\varepsilon > 0$ if and only if $y = b$. This contract violates Kate's incentive constraint and incentivizes her to bribe the appraiser in state g. If ε is sufficiently small, Susie's conjecture rationalizes Bruce's offer since the conjectured contract between Bruce and Kate guarantees Bruce's preferred realization. By accepting Bruce's offer, Susie exposes herself to a negative externality imposed on her by Kate's bribe. Therefore, Susie rejects Bruce's offer.

Is there a multilateral contract that is both weakly stable and constrained-efficient? Weakly stable contracts exist. However, the existence of a contract that is both weakly stable and constrained-efficient depends on the fraction of agents who are able to manipulate the signal. This dependence is non-monotone. When a subset of agents is unable to manipulate the signal, the answer to this question is negative. Moreover, the difference between the maximal level of insurance attainable via a weakly stable contract and the level of insurance in a constrained-efficient contract can be large. If, however, all agents are able to manipulate the signal, every weakly stable contract is constrained-efficient.

We apply our results in three different risk-sharing settings: a joint venture that involves agents with different risk attitudes, a setting in which only one agent is exposed to a shock, and a re-insurance market, which is our main application. Re-insurance instruments (e.g., catastrophe bonds) are used to transfer risk resulting from high-volume events from primary insurers to re-insurers or to the capital market. These instruments are often conditioned on state-dependent signals, in order to avoid moral hazard in underwriting and claim settlements (see Doherty, 1997). Such state-dependent signals include policy makers' actions (e.g., a declaration of a disaster), weather indices, and industry-loss indices.

We study a re-insurance market in which primary insurers who are exposed to a common shock transfer risk to an infinite group of re-insurers. Some of the primary insurers can potentially manipulate the signal. The re-insurers cannot manipulate the signal. We study the maximal coverage that the primary insurers can attain with weakly stable contracts that provide fair insurance and show that it is U-shaped in the number of potential manipulators. We obtain a closed-form solution and show that the negative effect on the level of coverage is of first-order magnitude. Moreover, even when manipulation becomes more costly such that constrained-efficient coverage approaches full insurance, the aforementioned effect does not disappear (i.e, it is bounded away from zero).

The paper proceeds as follows. We present the model in Section 2 and analyze it in Section 3. In Section 4 we present three applications. In Section 5 we discuss and relax some of our modeling assumptions. Section 6 concludes and discusses the related literature. In Appendix A we present and discuss manipulation games and in Appendix B we extend the model to allow for multilateral side-contracts. Proofs that are omitted from the main text appear in Appendix C.

2 The model

Let $I = \{1, ..., n\}$ be a set of agents and let $\Theta = \{1, 2\}$ denote the set of states. Each state $\theta \in \Theta$ is realized with probability p_θ and this is common knowledge among the members of I. Each agent i's preferences are represented by a strictly concave vNM utility function.[2] Each agent $i \in I$ has a positive endowment denoted by $w_i(\theta)$ in each $\theta \in \Theta$. Define $W_i := w_i(2) - w_i(1)$. We refer to W_i as agent i's *initial exposure*.

The state of nature is not contractible. Let $Y = \{1, 2\}$ be a set of signals. We use y to denote a typical element of Y. We assume that if no one manipulates the signal, it

[2]For ease of exposition, we assume the existence of risk-neutral agents in some of the applications. Also, whenever u_i is defined over $\mathbb{R}_{++}$, our convention is that $u_i(z) = -\infty$ for all $z \leq 0$.

matches the state of nature. The agents share risk using contracts that are contingent on the signal.

A multilateral contract $g^K : Y \to \mathbb{R}^n$ sets budget-balanced transfers among a group of agents $K \subseteq I$ as a function of the realized signal. We denote $g_i^K(y')$ the transfer received by $i \in K$ (according to g^K) in the case where[3] $y = y'$. For $i \notin K$, $g_i^K(y') := 0$. Define $R_i\left(g^K\right) := g_i^K(2) - g_i^K(1)$. We refer to $R_i(g)$ as agent i's *coverage* in g. For any two contracts $g^K, \bar{g}^{K'}$, we use $g^K + \bar{g}^{K'}$ to denote the summation over the transfers in the two contracts. That is, for each $y \in \{1, 2\}$, $\left(g^K + \bar{g}^{K'}\right)_i(y) := g_i^K(y) + \bar{g}_i^{K'}(y)$. We often focus on one multilateral contract that sums all the contracts between the members of I and we reserve g to describe this contract. We refer to $W_i + R_i(g)$ as agent i's *ex-post exposure*.

For ease of exposition, it is useful to use a different notation for bilateral contracts. A bilateral contract $b_{ij} : Y \to \mathbb{R}$ between i and j sets a transfer $b_{ij}(y)$ from j to i contingent on the realization of the signal. We write $b_{ij} + b_{kl}$ to denote a contract that sums the transfers made in the two bilateral contracts.

We assume that the signal is *manipulable*: each agent $i \in M \subseteq I$ can change its realization y from θ to $\theta' \neq \theta$ by paying a cost[4] $c > 0$. From now on, we denote the signal that results from the contract g in state θ by $y(g, \theta)$. We say that the contract g is incentive compatible (IC) if $|R_i(g)| \leq c$ for all $i \in M$. We assume that if g is IC, then $y(g, \theta) = \theta$ for each $\theta \in \{1, 2\}$. If g is not IC, then the set $PM(g) = \{i \in M : |R_i(g)| > c\}$ is not empty. We assume that, in this case, the lowest labeled agent $i \in PM(g)$ such that $i \in \arg\max_{i' \in PM(g)} |R_{i'}(g)|$ sets the signal such that $y(g, \theta) = 2$ if and only if $R_i(g) > 0$. Agent i pays a cost of c (0) if $y(g, \theta) \neq \theta$ ($y(g, \theta) = \theta$).

This assumption implies that if a contract is IC, no agent manipulates the signal. When some agents are incentivized to manipulate the signal (that is, when a contract is not IC), the agent who has the most coverage (in absolute value) is the one who decides on the signal's realization and pays for manipulation in the state in which he needs to manipulate the signal. We have in mind a case in which there are many random opportunities to manipulate the signal (the agents get random access to a regulator, or to the financial reports). All agents believe that the agent who receives the most coverage will manipulate the signal to his preferred realization whenever he gets the opportunity to do so, and therefore they abstain from incurring the manipulation cost.

[3]The results in the paper hold if the agents are allowed to burn money such that for each $y \in Y$, $\sum_{k \in K} g_k^K(y) \leq 0$.

[4]The symmetry assumption is discussed and relaxed in Section 5.

Since manipulation may take place in many different ways we use the aforementioned reduced form assumption and do not commit ourselves to a specific manipulation game. In Appendix A we discuss this assumption and show that it can be a result of an equilibrium in a manipulation game. We also present three games in which this assumption is not a result of an equilibrium. Nevertheless, we show that our main result holds when we replace our assumption with each of these three games.

We wish to emphasize that the agents manipulate the signal and not the state of nature. This implies that manipulations do not affect the agents' payoffs via their endowments. As an illustration, consider Example 1 and suppose that Kate and Bruce own equal shares of an asset. There are two ways for Kate to manipulate the appraiser's report. First, she can bribe the appraiser; this affects Bruce's payoff via the insurance contracts that he signed. Second, Kate can damage the asset such that the appraiser will report that the negative shock was realized. This type of manipulation affects Bruce's *endowments* and is not captured by the present model. We discuss state-contingent contracts and state manipulation in the concluding section.

A contract g is said to be individually rational (IR) for agent $i \notin PM(g)$ if

$$\begin{aligned} & p_1 u_i(w_i(1)) + p_2 u_i(w_i(2)) \\ \leq\ & p_1 u_i(w_i(1) + g_i(y(g,1))) + p_2 u_i(w_i(2) + g_i(y(g,2))) \end{aligned}$$

We say that g is IR if it is IR for all $i \in I$.

For each $i \in I$, we use $\succ_i$ to represent i's indirect preferences over contracts. For example, suppose that g is IC and g' is not IC: $PM(g') = \{j\}$ and $R_j(g') < 0$. Then, for $i \in I/\{j\}$, $g' \succ_i g$ if and only if

$$\begin{aligned} & p_1 u_i(w_i(1) + g_i(1)) + p_2 u_i(w_i(2) + g_i(2)) \\ <\ & p_1 u_i(w_i(1) + g'_i(1)) + p_2 u_i(w_i(2) + g'_i(1)) \end{aligned}$$

Note that in the expression in the RHS, agent i receives $g'_i(1)$ in both states since the signal is manipulated by j. For $i = j$, $g' \succ_i g$ if and only if:

$$\begin{aligned} & p_1 u_i(w_i(1) + g_i(1)) + p_2 u_i(w_i(2) + g_i(2)) \\ <\ & p_1 u_i(w_i(1) + g'_i(1)) + p_2 u_i(w_i(2) + g'_i(1) - c) \end{aligned}$$

Note that the manipulation cost c is taken into account only in state 2, when j needs to manipulate the signal.

In the model, there are four possible types of agents: manipulators or non-manipulators with positive or negative initial exposure. The following richness assumption guarantees that the economy is diverse in the sense that there exists at least one agent of each type.

Definition 1 *We say that the economy satisfies richness if there are two agents* $i, i' \in M$ *such that* $W_i > 0 > W_{i'}$ *and two agents* $j, j' \notin M$ *such that* $W_j > 0 > W_{j'}$*.*

Note that richness implies that $|I| > 3$. Moreover, richness rules out purely aggregate shocks (see, e.g., Example 1). In Section 5 we relax the richness assumption to account for purely aggregate shocks and analyze the case of $|I| = 3$.

3 Analysis

3.1 Constrained efficiency

Constrained efficiency is the notion of efficiency that we use throughout the paper. We use the term *constrained* to indicate that the notion of efficiency is based on the agents' indirect preferences that take manipulations into account.

Definition 2 *A contract* g *is constrained-efficient if it is IR and there exists no other contract* g' *such that* $g' \succ_i g$ *for some* $i \in I$ *and* $g' \succeq_i g$ *for all* $i \in I$*.*

We begin our analysis with a characterization of constrained-efficient contracts. The following simple result establishes that a contract g is IR only if it is IC. Note that proofs that do not appear in the main text are to be found in Appendix C.

Claim 1 *Suppose* g *is not IC. Then,* g *is not IR.*

Corollary 1 *A contract is constrained-efficient only if it is IC.*

The agents' primary goal is to reduce their exposure to the state of nature. Therefore, when c is relatively large, manipulation is irrelevant and the model collapses to a conventional exchange economy. The following non-triviality condition guarantees that the intersection between the set of Pareto-efficient contracts (in a conventional risk-sharing economy) and the set of constrained-efficient contracts is empty. That is, the agents' ability to manipulate the signal precludes Pareto-optimal risk-sharing.

Definition 3 *An economy is said to satisfy non-triviality if there exist two agents* $i, j \in M$ *such that* $W_i > c$ *and* $-W_j > c$.

Claim 2 *Suppose that the economy satisfies non-triviality. Then, the intersection between the set of Pareto-efficient contracts (i.e., the set of constrained-efficient contracts when* $M = \emptyset$*) and the set of constrained-efficient contracts (when* $M \neq \emptyset$*) is empty.*

We now establish another property of constrained-efficient contracts in economies that satisfy richness and non-triviality.

Proposition 1 *Suppose that the economy satisfies richness and non-triviality. Then, in every constrained-efficient contract* g *there is an agent* $i \in M$ *such that* $|R_i(g)| = c$ *and an agent* $j \notin M$ *such that* $sign(R_j(g)) = sign(R_i(g))$.

Proof. In a constrained-efficient contract g, if there are two agents i, j such that $W_i + R_i(g) > 0 > W_j + R_j(g)$, then $R_i(g) = -c$ or $R_j(g) = c$. Otherwise, there is a contract b_{ij} such that $b_{ij}(1) > 0 > b_{ij}(2)$ and its addition to g makes both i and j better off without violating their incentive constraints. The assumption that the economy satisfies non-triviality implies that in any constrained-efficient contract there is an agent $i \in M$ such that $|R_i(g)| = c$. Otherwise, there are two agents $m, m' \in M$ such that $|R_m(g)|, |R_{m'}(g)| < c$, and $W_m + R_m(g) < 0 < W_{m'} + R_{m'}(g)$.

Assume without loss of generality that there is an agent $m \in M$ such that $-R_m(g) = g_m(1) - g_m(2) = c$. If there exists an agent $j \notin M$ such that $R_j(g) < 0$, the proposition is proven. Assume that this is not the case. By richness, there are two agents $i, i' \notin M$ such that $W_i < 0 < W_{i'}$. By assumption $R_{i'}(g) \geq 0$. Therefore, $W_{i'} + R_{i'}(g) > 0$. By non-triviality, there is an agent $h \in M$ such that $W_h = w_h(2) - w_h(1) < -c$. By IR, $W_h + R_h(g) < 0$. Since $W_{i'} + R_{i'}(g) > 0$, constrained-efficiency implies that $R_h(g) = c$. Since $i, i' \notin M$, and $W_{i'} + R_{i'}(g) > 0$, constrained efficiency of g implies that $W_i + R_i(g) \geq 0$. It follows that $R_i(g) > 0$ and $R_h(g) = c$. ■

Equipped with these characteristics of constrained-efficient contracts, we now analyze decentralized risk-sharing.

3.2 Pairwise stability

In this subsection, the instability that follows from the agents' ability to write bilateral side-contracts is displayed. We view a multilateral contract g as *stable* if there exists no pair of agents who are better off writing a new bilateral contract (without having to withdraw from g). This notion of stability is inspired by Jackson and Wolinsky (1996).

Definition 4 *A contract g is pairwise stable if there exists no contract b_{ij} such that $g + b_{ij} \succ_i g$ and $g + b_{ij} \succ_j g$.*

Note that in Jackson and Wolinsky's notion of pairwise stability, a pair of agents is either connected to each other or not. That is, the conventional definition of pairwise stability refers to binary links. The present notion of stability is different from that of Jackson and Wolinsky since here bilateral contracts are vectors that specify budget-balanced transfers between contracting agents.

The following proposition establishes that IR pairwise stable contracts do not exist in economies that satisfy non-triviality and richness. In particular, in such economies, there exists no contract g that is both constrained-efficient and pairwise stable.

Proposition 2 *Suppose that non-triviality and richness are satisfied. There exists no multilateral contract g that is both IR and pairwise stable.*

Proof. Consider an arbitrary contract g and distinguish between two different cases: either g is constrained-efficient or not. Suppose g is constrained-efficient. By Proposition 1, there is an agent $i \in M$ such that $|R_i(g)| = c$ and an agent $j \notin M$ such that $sign(R_j(g)) = sign(R_i(g))$. Assume without loss of generality that $R_i(g) = c$. Consider a contract b_{ij} such that $b_{ij}(2) = \varepsilon > 0 = b_{ij}(1)$, that is, a contract in which agent j pays agent i an amount ε if and only if the signal is realized to be 1. Note that $R_i(g + b_{ij}) = c + \varepsilon > c$. It follows that $PM(g + b_{ij}) = \{i\}$ and $y(g + b_{ij}, \theta) = 2$ for each $\theta \in \{1, 2\}$. Since $R_j(g) > 0$, if ε is sufficiently small, $g + b_{ij} \succ_j g$ and $g + b_{ij} \succ_i g$.

To complete the proof we need to show that if g is not constrained-efficient, it cannot be both IR and pairwise stable. Assume by negation that g is IR, pairwise stable, and not constrained-efficient. By non-triviality, there are two agents i,$j \in M$ such that $W_i > c > -c > W_j$. The assumption that g is IR implies that $|R_i(g)|, |R_j(g)| \leq c$. Pairwise stability implies that either $R_i(g) = -c$ or $R_j(g) = c$. Otherwise, there is a contract between i and j that makes both of them better off without violating their incentive constraints. Assume without loss of generality that $R_i(g) = -c$. If there exists an agent k such that $R_k(g) < 0$, then there is a contract b_{ik} such that $b_{ik}(1) > 0 = b_{ik}(2)$ that violates the pairwise stability of g. That is, there is a contract that violates the pairwise stability of g, in which k incentivizes i to manipulate the signal from 2 to 1 by paying him a small amount if and only if the signal is realized to be 1. It follows that $R_h(g) \geq 0$ for all $h \in I / \{i\}$.

By richness, there are two agents $k, k' \notin M$ such that $W_k > 0 > W_{k'}$. Since $R_k(g) \geq 0$, $W_k + R_k(g) > 0$. Since $k, k' \notin M$, the pairwise stability of g and the inequality

$W_k + R_k(g) > 0$ imply that $W_{k'} + R_{k'}(g) \geq 0$. Thus, $R_{k'}(g) > 0$. Since $W_j < -c$, IR implies that $W_j + R_j(g) < 0$. Either there exists a legitimate coinsurance contract b_{jk} between j and k that violates the pairwise stability of g, or $R_j(g) = c$. Since $R_j(g) = c$ and $R_{k'}(g) > 0$, there exists a contract $b_{jk'}$ such that $b_{jk'}(2) = \varepsilon > 0 = b_{jk'}(1)$ (that is, a contract in which k' incentivizes j to manipulate the signal from 1 to 2 by paying him an amount ϵ if and only if the signal is realized to be 2) that violates the pairwise stability of g by making both j and k' better off. ■

The proof of Proposition 2 shows that when the economy satisfies non-triviality, then in a pairwise stable contract there is always at least one agent $i \in M$ who is indifferent between manipulating the signal and not doing so. This implies that any agent $j \in I$ such that $sign(R_i(g)) = sign(R_j(g))$ has the incentive to "break" i's indifference by paying him a small $\varepsilon > 0$ in one of the signal's realizations.

3.3 Weak stability

Pairwise stability makes a strong implicit assumption: when a pair of agents (i, j) deviates by adding a contract b_{ij}, each of the deviating agents holds a belief that there are no other deviations that will make him worse off if he agrees to b_{ij}. This assumption follows from the fact that pairwise stability considers the addition of one contract at a time. This is particularly restrictive since the attractiveness of a contract (and, in particular, of taking part in a deviation) is affected by the existence of side-contracts between other agents. This is because the existence of these additional side-contracts may create an incentive to manipulate the signal for some of these agents. An agent i who takes part in a deviation from the prevailing multilateral contract may suspect that his counter-party to the deviation has an ulterior motive such as additional side-contracts (with other agents) that make i worse off if he agrees to take part in this deviation. In this subsection we develop a stability notion that relaxes this implicit assumption.

Let us consider a possible deviation b_{ij}. It can be viewed as if it were initiated by one of the two agents, say, i. The question that naturally arises is what are j's beliefs about other contemporaneous deviations that i might have initiated. Pairwise stability includes an implicit assumption that j believes that i did not initiate any contemporaneous deviation b_{ik} that makes j worse off if he agrees to i's offer. Before we relax this assumption, we present an example in which healthy suspicion toward i's motivation is relevant.

Example 2 *Let* $I = \{1, ..., 8\}$*,* $M = \{1, 2, 3, 6, 7, 8\}$*, and* $c < 1$*. The following table* *summarizes the agents' initial exposure and the coverage received in a contract g.*

agent	1	2	3	4	5	6	7	8
W_i	1	1	1	1	-1	-1	-1	-1
$R_i(g)$	$-c$	$-c$	$-c$	0	0	c	c	c

We present two deviations that violate the pairwise stability of g. The first deviation decreases the exposure (in absolute value) of agents 4 and 5 (i.e., it is a legitimate co-insurance contract). We show that when this deviation is initiated by agent 5 (4), agent 4 (5) has reason to suspect that agent 5 (4) signed an additional contract with a third agent $k \in I/\{4,5\}$, thereby incentivizing k to manipulate the signal.

Since $W_4 > 0 > W_5$, there is a contract b_{45}, such that $b_{45}(1) > 0 > b_{45}(2)$, that reduces both agent 4's and agent 5's ex-post exposure (in absolute value) and is profitable to both of them. Suppose agent 4 initiates such a deviation b_{45}. Agent 5 might suspect that agent 4 has initiated another deviation b_{34} such that $b_{34}(1) = \varepsilon > 0 = b_{34}(2)$, that is, a deviation in which agent 4 incentivizes agent 3 to manipulate the signal from 2 to 1 by paying him an $\varepsilon > 0$ if and only if the realized signal is 1. If ε is sufficiently small, then $g + b_{34} + b_{45} \succ_4 g + b_{45}$ and $g + b_{34} + b_{45} \succ_4 g + b_{34}$. Agreeing to b_{45} exposes agent 5 to a negative externality imposed by agent 3's manipulation of the signal (given b_{34}). Note that $g + b_{34} \succ_5 g + b_{34} + b_{45}$ (that is, agent 5 is worse off agreeing to b_{45}).

We now present a second deviation in which agents 6 and 7 write a contract with the intention that agent 7 will manipulate the signal. Agent 7 suspects that agent 6 has an additional side-contract with agent 8. The conjectured contract between agents 6 and 8 incentivizes agent 6 to manipulate the signal himself. Agent 7's suspicion is that agent 6 is using their side-contract to make agent 7 manipulate the signal and pay the manipulation cost instead of agent 6 having to manipulate the signal and pay the cost himself.

Specifically, consider a deviation b_{76} initiated by agent 6, such that $b_{76}(2) = \varepsilon > 0 = b_{76}(1)$. That is, agent 6 makes an offer to agent 7 that is supposed to break 7's indifference and incentivize him to manipulate the signal from 1 to 2. Agent 7 may suspect that agent 6 also initiated a deviation b_{68} such that $b_{68}(y) = b_{76}(y)$ for each $y \in \{1,2\}$. That is, agent 6 made an offer to agent 8 to break his own indifference such that $R_6(g + b_{68}) > c$ and $y(g + b_{68}, \theta) = 2$ for each $\theta \in \{1,2\}$. If ε is small, then

$$
\begin{aligned}
& p_1 u_6 \left(w_6 \left(1\right) + g_6 \left(2\right) + \varepsilon - c\right) + p_2 u_6 \left(w_6 \left(2\right) + g_6 \left(2\right) + \varepsilon\right) \\
< \;\; & p_1 u_6 \left(w_6 \left(1\right) + g_6 \left(2\right)\right) + p_2 u_6 \left(w_6 \left(2\right) + g_6 \left(2\right)\right)
\end{aligned}
$$

The LHS is agent 6's payoff under $g + b_{68}$ and the RHS is his payoff under $g + b_{76} + b_{68}$. Since $g + b_{76} + b_{68} \succ_6 g + b_{68}$, the observed contract b_{76} can be rationalized by the conjectured contract b_{68}. Note that the realized signal is identical in both cases. However, the identity of the agent who pays for the manipulation is different.

Let us study agent 7's considerations. Observe that $y\left(g + b_{68}, \theta\right) = 2$ in both states since 6 manipulates the signal. Agreeing to agent 6's offer does not change the realization of the signal (in both states) since $R_7\left(g + b_{76}\right) > R_6\left(g + b_{68} + b_{76}\right) = c$. However, it changes the identity of the agent who has to pay for the manipulation in state 1. Agreeing to b_{76} makes agent 7 pay the cost of manipulation instead of agent 6 paying this cost. Since ε is assumed to be small relative to c, agent 7 is worse off agreeing to agent 6's offer.

We now present a notion of stability that takes into account the suspicion motive presented above. Our notion of stability involves suspicion towards agents who break the norm and initiate deviations from the prevailing multilateral contract. Given a contract g and an offer b_{ij} made by i, $\beta_j\left(b_{ij}, g\right) = \left(b_{ik}\right)_{k \in I/\{i,j\}}$ denotes agent j's belief about the other offers that i has made to other agents (which have been accepted). We assume that agent j cannot observe deviations that do not include him. If $g + \beta_j\left(b_{ij}, g\right) + b_{ij} \prec_j g + \beta_j\left(b_{ij}, g\right)$, then j has an incentive to reject i's offer. In this case, we say that the deviation b_{ij} is *blocked* by $\beta_j\left(b_{ij}, g\right)$. We now elaborate on the beliefs that agent j is allowed to hold.

Definition 5 *A belief $\beta_j\left(b_{ij}, g\right)$ is consistent if $g + \beta_j\left(b_{ij}, g\right) + b_{ij} \succ_i g + \beta_j\left(b_{ij}, g\right)$.*

Agent j's belief about the other contracts signed by i, $\beta_j\left(b_{ij}, g\right)$, is consistent with the observed contract b_{ij} if the addition of b_{ij} to $g + \beta_i\left(b_{ij}, g\right)$ makes agent i better off. Note that a consistent belief may not exist. For example, agent j has no belief that is consistent with a contract b_{ij} in which i pays j the same positive amount in both realizations of the signal. These cases are of lesser interest since agent i, being rational, never makes an offer that cannot be rationalized.

Definition 6 *A contract g is weakly stable if, for each $i \in I$ and contract b_{ij} such that $g + b_{ij} \succ_i g$, there exists a consistent belief $\beta_j\left(b_{ij}, g\right)$ such that $g + \beta_j\left(b_{ij}, g\right) \succ_j g + \beta_j\left(b_{ij}, g\right) + b_{ij}$.*

Observe that a deviation consists of a contract and an agent who initiates it. The same contract is treated differently when the identity of its initiator is different. It can be the case that a contract b_{ij} is blocked by a belief $\beta_j(b_{ij}, g)$ but is not blocked by any belief $\beta_i(b_{ij}, g)$. That is, j (i) has an (no) incentive to reject b_{ij} when it is offered by i (j). Weak stability of g requires that the side-contract b_{ij} is blocked by a belief $\beta_i(b_{ij}, g)$ and by a belief $\beta_j(b_{ij}, g)$.

Let us consider the logic of weak stability. An agent j who receives an offer from agent i needs to form some belief about other offers that i might have initiated. Let us think of these offers as i's type. Consistency implies that j is restricted to holding a belief that i's type is one that may potentially benefit from such an offer. A contract g is weakly stable if it is a best response for j to reject i's offer, given the restriction on j's beliefs. If this is the case, then it is a best response for i not to initiate the deviation in the first place.

Cho and Kreps (1987) provide an intuitive criterion to determine the stability of equilibria in signaling games (sender-receiver games) based on restrictions on out-of-equilibrium beliefs. Our stability concept employs the same logic. The intuitive criterion consists of two steps. The first step restricts the beliefs of a receiver of an out-of-equilibrium message; he must hold a belief that the message sender's type is one that can benefit from the message. The second requirement for an equilibrium to survive the intuitive criterion is that, under this restriction on the receiver's beliefs, he have some best response to this belief that makes it not beneficial to send the out-of-equilibrium message in the first place.

Note that an agent i who receives an offer to deviate from the prevailing contract forms beliefs that take into account the initiator of the deviation's incentives, and not the incentives of other agents. If i's beliefs are restricted by the other agents' considerations (for example, if he is not allowed to believe that agent k agreed to a contract that can render k worse off), then our ability to disqualify deviations is restricted. Such a concept would be stronger than weak stability. Since our main results are negative, the weaker our stability concept, the more persuasive they are. Therefore, we use a relatively weak concept and view the positive results as an upper bound for the level of insurance that is attainable.

The stability notion does not include the possibility of canceling a signed contract. Allowing the agents to cancel previously signed contracts allows for more deviations than the present stability notion. However, it does not add new beliefs that can disqualify deviations. To see that, consider agent j who receives an offer to sign a bilateral contract with agent i and rejects it based on a conjecture that i contemporaneously canceled a contract he previously signed with k. Since weak stability does not impose any restriction on j's beliefs about agent k's considerations, there is a bilateral contract between i and k that can be conjectured by j and have the same effect as the cancellation of the contract between i and k. We do not allow for cancellation of contracts since it does not change the main results and we prefer our stability notion to be weaker in order to emphasize the negative results.

3.3.1 Existence

We now establish the existence of weakly stable contracts. The existence proof relies on the following lemma.

Lemma 1 *Let g be an IR contract. Suppose that the economy satisfies richness and consider a contract b_{ij} such that $b_{ij}(y) > 0$ for some $y \in \{1,2\}$, offered by $i \in I$. There exists a consistent belief $\beta_j(b_{ij}, g)$ that blocks b_{ij}.*

Proof. Let g be an arbitrary IR contract. Without loss of generality, consider a contract b_{ij}, offered by i, such that $b_{ij}(2) > 0$, that is, a contract in which j pays i a strictly positive amount if the signal is realized to be 2. Suppose $i \in M$. Fix a belief $\beta_j(b_{ij}, g) = b_{ik}$ such that $k \notin M$. If $R_i(b_{ik}) = b_{ik}(2) - b_{ik}(1)$ is sufficiently high, $y(g + b_{ik} + b_{ij}, \theta) = 2$ for each $\theta \in \{1,2\}$. Since $b_{ij}(2) > 0$, $g + b_{ik} + b_{ij} \succ_i g + b_{ik}$. It follows that $\beta_j(b_{ij}, g)$ is consistent and $g + \beta_j(b_{ij}, g) + b_{ij} \prec_j g + \beta_j(b_{ij}, g)$.

To complete the proof, suppose that $i \notin M$. By richness, $M/\{j\}$ is not empty. Fix a belief $\beta_j(b_{ij}, g) = b_{ik}$, $k \in M/\{j\}$. For a sufficiently large $R_k(b_{ik}) = b_{ik}(1) - b_{ik}(2)$, $R_k(g + b_{ik}) > \max\{|R_j(g + b_{ij})|, c\}$ such that $y(g + b_{ij} + b_{ik}, \theta) = 2$ for each $\theta \in \{1,2\}$. Since $b_{ij}(2) > 0$, $\beta_j(b_{ij}, g)$ is consistent and $g + \beta_j(b_{ij}, g) + b_{ij} \prec_j g + \beta_j(b_{ij}, g)$. ■

Lemma 1 shows that agent j rejects any contract in which he is supposed to pay agent i in one of the realizations. As a result, bilateral side-contracts signed with the intention that the contracting parties co-insure each other cannot undermine the weak stability of a multilateral contract. This is because in any co-insurance contract b_{ij}, it must be that $b_{ij}(2) > 0 > b_{ij}(1)$ or $b_{ij}(1) > 0 > b_{ij}(2)$. The only deviations that can violate the weak stability of a contract g are those that incentivize one of the deviating counter-parties to manipulate the signal. Moreover, these deviations include one agent j who pays his counter-party, agent i, a positive amount in both of the signal's realizations (i.e., $b_{ij}(2), b_{ij}(1) \geq 0$).

Lemma 1 demonstrates the permissiveness of weak stability: there are no restrictions on j's beliefs about k's considerations. Moreover, there are no restrictions about

j's beliefs about i's considerations w.r.t. b_{ik}. A similar result could be obtained even if one imposes one of these two restrictions.

Proposition 3 *Consider an economy that satisfies richness. There exists a weakly stable contract g.*

Proof. We consider a degenerate contract g and show that it is weakly stable. We partition the possible deviations that an arbitrary agent $i \in I$ may initiate into two: deviations b_{ij} such that $b_{ij}(y) > 0$ for some $y \in \{1,2\}$ and deviations such that $b_{ij}(y) \leq 0$ for each $y \in \{1,2\}$. Since $g_i(1) = g_i(2) = 0$, we do not need to worry about deviations of the latter type: for every such offer b_{ij}, i receives less than $u_i(w_i(\theta))$ in each state $\theta \in \{1,2\}$. By Lemma 1, deviations of the former type are blocked by some consistent belief. ■

Note that Proposition 3 relies only on the fact that $|M| > 1$ and that there exists an agent $i \notin M$. It will be shown that existence of weakly stable contracts for the special case of $I = M$ follows from the positive result of Proposition 5.

3.3.2 Main results

We now present the main negative result of the paper. It establishes that when the economy satisfies non-triviality and richness, it is impossible to write a multilateral risk-sharing contract that is both weakly stable and constrained-efficient. The following lemma is the cornerstone of the proof. It highlights one deviation that can never be blocked by a consistent belief.

Lemma 2 *Consider an IR contract g, and two agents $m \in M$, $i \notin M$. Every deviation b_{mi} such that $b_{mi}(2)$, $b_{mi}(1) \geq 0$, and $g + b_{mi} \succ_i g$ is not blocked by any consistent belief $\beta_m(b_{mi}, g)$.*

Proof. Let g be an arbitrary multilateral IR contract. Consider a bilateral contract b_{mi} such that $b_{mi}(2)$, $b_{mi}(1) \geq 0$, and $g + b_{mi} \succ_i g$, that is, a contract in which i pays m a positive amount in both realizations. Since g is IR and $b_{mi}(2), b_{mi}(1) \geq 0$, the fact that $g + b_{mi} \succ_i g$ implies that b_{mi} must incentivize m to manipulate the signal. It follows that $|R_m(g + b_{mi})| > c$.

Assume, without loss of generality, that $R_m(g + b_{mi}) > c$ and consider an arbitrary consistent belief $\beta_m(b_{mi}, g)$. We now show that $\beta_m(b_{mi}, g)$ does not block b_{mi}. Since $b_{mi}(2), b_{mi}(1) \geq 0$, consistency of $\beta_m(b_{mi}, g)$ implies that m manipulates the signal given the contract $g + \beta_m(b_{mi}, g) + b_{mi}$ so that $y(g + \beta_m(b_{mi}, g) + b_{mi}, \theta) = 2$ for each $\theta \in \{1, 2\}$. Also, consistency implies that the contract b_{mi} changes the realized signal in one of the states. Therefore, $y(g + \beta_m(b_{mi}, g), 1) = 1$. Agent m is not worse off in state 2 since he does not incur any cost and $y(g + \beta_m(b_{mi}, g) + b_{mi}, 2) = 2$. In state 1 agent m receives a payoff of $u_m(w_i(1) + g_m(2) + b_{mi}(2) - c) > u_m(w_i(1) + g_m(1) + b_{mi}(1)) > u_m(w_i(1) + g_m(1))$. Therefore, $g + \beta_m(b_{mi}, g) + b_{mi} \succ_m g + \beta_m(b_{mi}, g)$. ∎

We now present the main result of the paper. It utilizes the characterization of constrained-efficient contracts obtained in Proposition 1 and shows that such a contract cannot be weakly stable.

Proposition 4 *Consider an economy that satisfies richness and non-triviality. There exists no contract that is both constrained-efficient and weakly stable.*

Proof. By Proposition 1, richness and non-triviality imply that in every constrained-efficient contract g there is an agent $i \in M$ such that $|R_i(g)| = |g_i(2) - g_i(1)| = c$ and an agent $j \notin M$ such that $sign(R_j(g)) = sign(R_i(g))$. Without loss of generality, assume that $R_i(g) = c$. Consider a contract b_{ij} such that $b_{ij}(2) > b_{ij}(1) > 0$. This contracts violates i's incentive constraint and incentivizes him to manipulate the signal from 1 to 2. If $b_{ij}(2)$ is sufficiently small, $g + b_{ij} \succ_j g$. By Lemma 2, there exists no consistent belief $\beta_i(b_{ij}, g)$ such that $g + \beta_i(b_{ij}, g) + b_{ij} \prec_i g + \beta_i(b_{ij}, g)$. ∎

Proposition 4 establishes that even under the weak restrictions imposed by weak stability, multilateral trade further constrains the agents' ability to share risk. What if the economy does not satisfy richness? In particular, it is interesting to study an economy in which all agents are able to manipulate the signal. Proposition 5 provides us with the answer to this question, which is the main positive result of the paper. The proposition follows directly from the following lemma.

Lemma 3 *Suppose that $n > 3$ and $|M| \geq 3$. Let g be an IR contract and consider two agents $m \in M$ and $m' \in M$. Any deviation $b_{m'm}$ such that $g + b_{m'm} \succ_m g$ is blocked by a consistent belief $\beta_{m'}(b_{m'm}, g)$.*

The next proposition follows directly from Lemma 3

Proposition 5 *Suppose that $n > 3$ and $M = I$. Each constrained-efficient contract g is weakly stable.*

Note that the proposition relies on the assumption that $n > 3$. The result is true for $n = 3$ as well. However, the proof is different. We present it in the discussion in Section 5.

The following corollary characterizes weak stability by summarizing the results obtained in Lemmata 1 ,2 , and 3. We will utilize it in the applications section.

Corollary 2 *Let* $n > 3$, $|M| > 2$, *and* g *be an arbitrary IR contract. Any contract* b_{mi} *such that* $m \in M$, $i \in I$, $b_{mi}(1), b_{mi}(2) \geq 0$, *and* $g + b_{mi} \succ_i g$ *violates the weak stability of* g. *Any other contract does not violate the weak stability of* g.

Proof. The above-described deviation violates the weak stability of g by Lemma 2. By Lemma 3, any deviation that includes two members of M cannot undermine the weak stability of g. When $M \neq I$ there must be at least one agent $i \notin M$. Since the only use that Lemma 1 makes of richness is the fact that it implies the existence of an agent $i \notin M$, we can use the lemma to disqualify every deviation b_{ij} such that $b_{ij}(2) > 0 > b_{ij}(1)$. ■

The fact that a particular weakly stable contract is not constrained-efficient does not imply that *all* of the agents are underinsured (relative to the coverage they obtain in a constrained-efficient contract). This is demonstrated in the following example.

Example 3 *Let* $I = \{1, ..., 6\}$, $c = 0.5$, $p_1 = 0.5$, *and* $M = \{1, ..., 4\}$. *Suppose that* $u_i(x) = -\exp(-\alpha x)$ *for all* $i \in I$. *The following table summarizes the agents' initial exposure:*

agent	1	2	3	4	5	6
W_i	1	1	1	-1	-1	-1

Every constrained-efficient contract g *provides the following coverage:*

agent	1	2	3	4	5	6
$R_i(g)$	-0.5	-0.5	-0.5	0.5	0.5	0.5

.

To see that, observe agent i*'s marginal rate of substitution between consumption in both states and note that it depends only on* $W_i + R_i(g)$. *Therefore, if* $W_i + R_i(g) \neq W_j + R_j(g)$, *either* $|R_i(g)| = c$ *or* $|R_j(g)| = c$. *By Corollary 2,* g *is not weakly stable. We now demonstrate the existence of a contract that is weakly stable, not constrained-efficient, and under which agents* 5 *and* 6 *receive more coverage than they receive in* g:

agent	1	2	3	4	5	6
$R_i(g')$	-0.5	-0.5	-0.5	0	0.75	0.75

Let us show that g' is weakly stable. By Corollary 2, contracts that include two agents $i, j \in \{1, 2, 3, 4\}$ cannot undermine the weak stability of g'. A deviation b_{56} cannot undermine the weak stability of g' since $W_5 + R_5(g') = W_6 + R_6(g')$ such that the agents have no mutual insurance motive and they cannot manipulate the signal. By Corollary 2, deviations b_{ij} such that $b_{ij}(1) > 0 > b_{ij}(2)$ do not undermine the weak stability of g' for any $i, j \in \{1, ..., 6\}$. Also, any contract in which agent 5 (6) pays agent 4 $b_{45}(2) > 0.5$ ($b_{46}(2) > 0.5$) with the intention that agent 4 will manipulate the signal, is not beneficial for agent 5 (6). It follows that g' is weakly stable.

In the example, agents 1, 2, and 3 can provide a total coverage of 1.5 dollars. The fact that the agents can add side-contracts to g' constrains agent 4's ability to insure himself. Roughly speaking, agent 5 and agent 6 benefit from this as they obtain more insurance w.r.t. the level of insurance they obtain via g. Note that the sum of the agents' ex-post exposure (in absolute value) in g' is identical to their ex-post exposure in g. In general (under any assumption about the agents' preferences), the summation of the agents' ex-post exposure is minimized in constrained-efficient contracts, but there may be other contracts (as shown in the example) that induce the same summation.

4 Applications

4.1 Re-insurance

We study a re-insurance market in which primary insurers who are exposed to a negative shock transfer risk to an infinite group of re-insurers. The risk-sharing contracts are conditioned on a state-dependent signal such as a policy maker's action or an industry-loss index. One possible interpretation of manipulation in this section is lobbying in order to influence some policy maker's action that the firms contract upon. For example, a deceleration on a disaster. The literature on lobbying (for a textbook treatment see Grossman and Helpman, 2001) with complete information assumes that the agents play a contribution game (Bernheim and Whinston, 1986). Using such a game as a manipulation game would change the present paper's specification, but the results obtained in this section would not change.

Our economy is composed of two disjoint sets: a set of insurers L and a set of re-insurers E. We assume that $M \subseteq L$. That is, only the insurers are able to manipulate the signal. We assume that all of the insurers are exposed to the same shock: for each $i \in L$, $w_i(1) = w_h > w_l = w_i(2)$, $w := w_h - w_l$. We further assume that $u_i = u$ for each $i \in L$ and that the cardinality of E is large relative to that of L such that the members of E can absorb all the risk in the economy. We study the limit case of $\frac{|E|}{|L|}$ tends to infinity. To avoid frictions arising from the discreteness of L, we assume that there are many insurers and denote the share of manipulators $|M| / |L|$ by α. For ease of exposition, we assume that the re-insurers are risk-neutral.

We are interested in the level of coverage $\sum_{i \in L} R_i(g)$ that the primary insurers can attain. In a constrained-efficient contract each $m \in M$ receives a coverage of c dollars and each $i \in L/M$ receives a coverage of w dollars. The average coverage is $\alpha c + (1 - \alpha) w$. By Corollary 2, if $\alpha \in (0, 1)$, then there exists no contract that is both constrained-efficient and weakly stable. This is because in such a contract each $m \in M$ is indifferent to manipulating the signal and each $i \in L/M$ receives strictly positive coverage. As a result, in IR weakly stable contracts, the agents are underinsured.

We are interested in the magnitude of the effect (on the level of attainable coverage) generated by the ability to write side-contracts. In particular, we are interested in the case where $\frac{c}{w}$ is close to 1. That is, manipulation becomes costly and constrained-efficient risk-sharing is close to full insurance. We wish to find out whether the negative effect of the ability to write side-contracts vanishes in this case.

We restrict out attention to *fair* insurance contracts. A contract g is fair if $p_1 g_i(1) + p_2 g_i(2) = 0$ for each[5] $i \in I$. This enables us to study the maximal coverage attainable for primary insurers who face re-insurers in a competitive setup. Note that constrained-efficient insurance is attainable using contracts that provide fair insurance. Technically, the restriction to fair insurance contracts simplifies the analysis as agent i's willingness to pay in order to guarantee his preferred realization (of the signal) is pinned down by his coverage $R_i(g)$. An equivalent way of simplifying the analysis is to assume constant absolute risk-aversion preferences. Since we are interested in the level of coverage as an approximation to welfare, we do not allow for overinsurance. Therefore, we restrict ourselves to contracts g such that $R_i(g) \leq w$ for each $i \in L$.

We start our analysis by characterizing the maximal average coverage that the primary insurers can attain using a weakly stable fair contract as a function of the share of manipulators α.

[5]We can obtain the same results by restricting attention to contracts in which $p_1 g_i(1) + p_2 g_i(2) < 0$ for each i such that $g_i(1) < g_i(2)$, that is, contracts in which agents who are covered pay a premium.

Proposition 6 *Suppose $M \geq 3$. There exists an α^* such that the maximal average coverage that can be obtained via a weakly stable contract that provides fair insurance is increasing (decreasing) in α for each $\alpha > \alpha^*$ ($\alpha < \alpha^*$).*

Proof. By Corollary 2, the only deviations with the potential to undermine the weak stability of g are those with one agent $i \notin M$ and one agent $m \in M$. In these deviations agent i pays agent m a positive amount in both realizations and incentivizes him to manipulate the signal. Consider an arbitrary IR contract g. Let us denote agent i's willingness to pay in order to guarantee his preferred realization y by $x_i^y(g)$. Formally,

$$\begin{aligned} & p_1 u(w_h + g_i(1)) + p_2 u(w_l + g_i(2)) \qquad (1) \\ = \; & p_1 u(w_h + g_i(y) - x_i^y(g)) + p_2 u(w_l + g_i(y) - x_i^y(g)) \end{aligned}$$

Each side-contract b_{mi} in which agent i pays agent m $z_y < x_i^y(g)$ when y is realized (and $z \in [0, z_y)$ otherwise) such that m's incentive constraint is violated, breaks the weak stability of g. By Corollary 2, these are the only side-contracts that violate the weak stability of g. By the assumption that the E is large, the coverage obtained in g (by the primary insurers) can be divided between the re-insurers such that $R_i(g)$ is arbitrarily small for each $i \in E$. As a result, $x_i^y(g)$ can be set to be sufficiently small to prevent deviations in which $i \in E$ pays $m \in M$ in order to incentivize m to manipulate the signal.

We now write the simplified maximization problem.

$$\max_{g_i(1), g_i(2) | i \in L} \sum_{i \in L} g_i(2) - g_i(1) \qquad (2)$$

$$\begin{aligned} & s.t \\ c \; & \geq \; \max_{m \in M} \{g_m(2) - g_m(1)\} + \max_{i \in L/M} x_i^2(g) \\ w \; & \geq \; \max_{i \in L} \{g_i(2) - g_i(1)\} \end{aligned}$$

First, note that we omit the IR constraints from the description of the problem. Our restriction to fair insurance contracts implies that as long as $R_i(g) \in [0, w]$ and g is IC, agent $i \in L$ is better off signing this contract. The first constraint guarantees the weak stability of g. Observe that it must be binding. Otherwise, there is an agent $m \in M$ such that $g_m(2) - g_m(1) < c < w$ who can receive more coverage without violating this constraint. Moreover, at the optimum, $g_m(2) - g_m(1) = g_{m'}(2) - g_{m'}(1)$ for each $m, m' \in M$. Otherwise, one can either increase m's coverage without violating the constraint or one can increase m''s coverage without violating the constraint. Denote by R_M the coverage received by each $m \in M$. The same argument can be applied to show that $g_i(2) - g_i(1) = g_{i'}(2) - g_{i'}(1)$ for each $i, i' \in L/M$. Denote by R_L the coverage received by each $i \in L/M$.

We now study how R_L changes with R_M. Consider agent $i \in L/M$. Recall that agent i receives fair insurance. Plugging fair insurance into equality (1), using the implicit function theorem, and deriving by $x_i^2(g)$, we obtain that $g_i(2) - g_i(1)$ is concave in $x_i^2(g)$ if and only if

$$\left(u'(w_h + g_i(1)) - u'(w_l + g_i(2))\right)\left[p_1 u''\left(w_h + g_i(2) - x_i^2(g)\right) + p_2 u''\left(w_l + g_i(2) - x_i^2(g)\right)\right] \geq 0$$

The above inequality holds since u is concave and $g_i(2) - g_i(1) \leq w$. If $g_i(2) - g_i(1) < w$, it holds with strict inequality. Since $x_i^2(g)$ is decreasing and linear in R_M, R_L is decreasing and convex in R_M. Since we maximize a linear combination of R_L and R_M subject to a convex constraint, the solution to the problem is unique and R_M is weakly increasing in α. Denote the solutions to the maximization problem for a specific α by R_M^α and R_L^α. If $R_M^\alpha > R_L^\alpha$ ($R_M^\alpha < R_L^\alpha$), then increasing (decreasing) α can only increase the maximal level of coverage that is attainable. By continuity, there exists an $\alpha \in (0,1)$ for which $R_M^\alpha > R_L^\alpha$ ($R_M^\alpha < R_L^\alpha$). ■

Proposition 6 shows that the maximal coverage that can be obtained using fair insurance contracts is U-shaped in the number of agents who can manipulate the signal. It implies that when it is easier to manipulate the signal in the sense that more agents are capable of doing so, the level of coverage may increase. When it is easier to manipulate the signal in the sense that c is close to w, the level of coverage increases. This is a result of the first constraint of Problem (2). In the sequel we shall show that the level of coverage is bounded away from full insurance when c approaches w. That is, the negative effect of the ability to add side-contracts on multilateral risk-sharing does not disappear when constrained-efficient coverage is close to full coverage.

Proposition 6 utilizes the fact that $x_i^y(g)$ is pinned down by $R_i(g)$. This is a result of our assumption that the re-insurance market is competitive such that $p_1 g_i(1) + p_2 g_i(2) = 0$. In general, $x_i(g)$ is not pinned down by $R_i(g)$ because absolute risk aversion affects the agents' willingness to pay for manipulation. As a result, R_L is a function of both R_M and α. Under constant absolute risk-aversion preferences, $x_i^y(g)$ is pinned down by $R_i(g)$. In this case, the maximal average coverage is U-shaped in α.

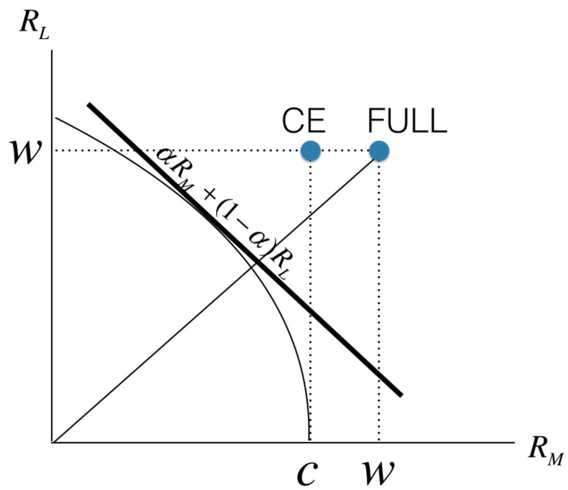

Figure 1: Problem (2) for a given α.

We now illustrate the result obtained in Proposition 6 by solving for the maximal average coverage that the agents can obtain facing a competitive re-insurance market for a specific set of parameters. We assume that $u_i(z) = \log(z)$ for each $i \in L$ and that the two states are equally likely. For ease of exposition, we assume that $w_l = 0$, $w_h = w$. In Proposition 6 we showed that it is without loss of generality to restrict attention to contracts in which each $i \in L/M$ ($i \in M$) receives the same payoffs R_L and R_M. Plugging the parameters into agent i's willingness to pay for manipulation of the signal that is given in (1):

$$x_i^2(g) = \frac{1}{2}\left(R_L + w - \sqrt{w^2 - R_L^2 + 2wR_L}\right)$$

At the optimum,

$$c = R_M + \frac{1}{2}\left(R_L + w - \sqrt{w^2 - R_L^2 + 2wR_L}\right) \tag{3}$$

Plugging expression (3) into the maximization problem given in (2), we obtain the following problem (note that we need to make sure that $R_M \geq 0$ because of IR).

$$\max_{R_L} \alpha\left(c - \frac{1}{2}\left(R_L + w - \sqrt{w^2 - R_L^2 + 2wR_L}\right)\right) + (1-\alpha)R_L$$

$s.t$

$R_L \leq w$

$R_M \in [0, c]$

The first-order condition of the above problem is

$$-\frac{1}{2}\alpha+\frac{1}{2}\alpha\frac{w-R_L}{\sqrt{w^2-R_L^2+2wR_L}}+(1-\alpha)=0$$

Rearranging, in an internal solution,

$$R_L=w\left(1-\sqrt{\frac{2\left(3-\frac{2}{\alpha}\right)^2}{1+\left(3-\frac{2}{\alpha}\right)^2}}\right)$$

Define $\Lambda:=1-\sqrt{\left(\frac{2\left(3-\frac{2}{\alpha}\right)^2}{1+\left(3-\frac{2}{\alpha}\right)^2}\right)}$. In an internal solution, the maximal average coverage is given by

$$\alpha\left(c-\frac{1}{2}w\left(\Lambda+1-\sqrt{1-\Lambda^2+2\Lambda}\right)\right)+(1-\alpha)\,w\Lambda$$

We now check whether $R_L\leq w$ and $R_M\geq 0$, that is, whether the problem has a corner solution. It is easy to see that $R_L\leq w$ if and only if $\alpha\leq\frac{2}{3}$. If $\frac{c}{w}\geq\frac{1}{2}\left(\Lambda+1-\sqrt{1+\Lambda^2+2\Lambda}\right)$, then[6] we have an internal solution for $\alpha\geq\frac{2}{3}$.

For $\alpha<\frac{2}{3}$ and $\frac{c}{w}\geq\frac{2-\sqrt{2}}{2}$, non-manipulators are fully insured. Substituting $R_L=w$ into constraint (3) we get that the maximal average coverage is

$$\alpha\left(c-\frac{1}{2}w\left(2-\sqrt{2}\right)\right)+(1-\alpha)\,w$$

If $\frac{c}{w}<\frac{2-\sqrt{2}}{2}$ and $\alpha<\frac{2}{3}$, manipulators are not insured (that is $R_M=0$) and non-manipulators receive coverage of $c+\sqrt{2wc-c^2}$ each. The maximal average coverage is

$$(1-\alpha)\left(c+\sqrt{2wc-c^2}\right)$$

This is also the maximal average coverage in the case that $\alpha>\frac{2}{3}$ and $\frac{c}{w}\leq\frac{1}{2}\left(\Lambda+1-\sqrt{1+\Lambda^2+2\Lambda}\right)$.

Note that when $\frac{c}{w}$ is low, then from the coverage maximization point of view, it is ideal if the manipulators have "skin in the game", that is, if $R_M<0$. This can happen only when the fair insurance assumption is relaxed. Observe that there is a trade off. Since $w>0$ and the agents are risk-averse, non-manipulators must subsidize this negative coverage. This is beneficial only when α is low since, in this case, there are only a few manipulators to subsidize.

[6] Note that $\frac{1}{2}\left(\Lambda+1-\sqrt{1+\Lambda^2+2\Lambda}\right)\leq\frac{2-\sqrt{2}}{2}$.

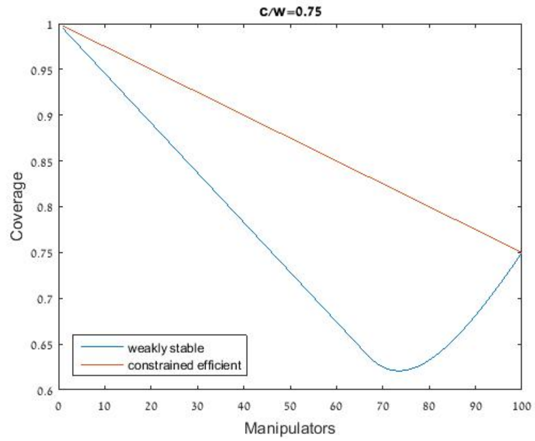

Figure 2: Attainable coverage for $\frac{c}{w} = 0.75$

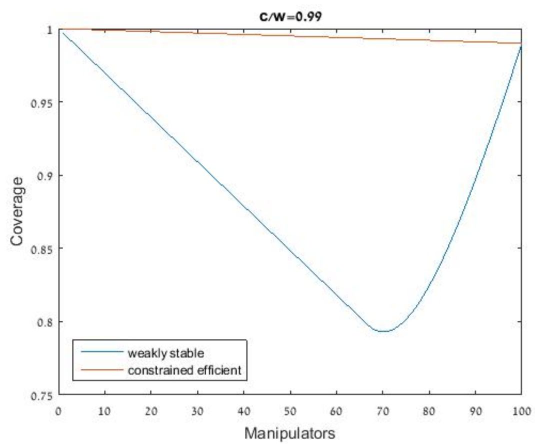

Figure 3: Attainable coverage for $\frac{c}{w} = 0.99$

The second extreme case is the case where $\frac{c}{w}$ approaches 1. That is, it is relatively hard to manipulate the signal. Observe that in this case the constrained-efficient level of insurance $\alpha c + (1 - \alpha) w$ is close to full insurance. However, the maximal coverage that the agents can attain in a weakly stable contract is bounded away from zero for $\alpha < 1$. For example, $\alpha = 0.5$ ($\alpha = 0.85$) implies a loss of 14.6 (13.4) percent w.r.t. the constrained-efficient coverage. Figure 2 (3) illustrates the agents' maximal insurance coverage as a function of the number of potential manipulators for $c = 0.75w$ ($c = 0.99w$) and $|L| = 100$. It compares it to the constrained-efficient level of insurance.

4.2 A private shock

In this subsection we consider a case in which there is only one agent who is exposed to a shock. We first assume that this agent is the only one who observes the shock's realization. The story we have in mind is that of an agent who owns an asset that can be damaged. The other agents are assumed to be insurance companies. The agents can transfer risk between them using contracts that are conditioned on the insured agent's report. The cost of reporting that a shock occurred when it did not in fact occur (when it occurred) is c (0). It represents the risk of getting caught lying. It is costless to report that the shock has not been realized since, in this case, there is no inspection by the insurance companies. This constitutes a relaxation of richness in two respects. First, we assume that only one agent is exposed to the shock. Second, it is assumed that this agent is the only one who can manipulate the signal. Let us summarize the agents' endowments (we assume that the negative shock is realized in state 2):

state/agent	k	1	2	...	$n-1$
1	$w+z$	w^1	w^2	...	w^{n-1}
2	w	w^1	w^2	...	w^{n-1}

In this case, the ability to write side-contracts does not restrict the agents' ability to share risk.

Claim 3 *Each constrained-efficient contract is pairwise stable.*

Proof. Let g be an arbitrary constrained-efficient contract. By IR, $R_k(g) = g_k(2) - g_k(1) \geq 0$. By the constrained efficiency of g, $R_i(g) \leq 0$ for each $I/\{k\}$. Otherwise, there are two agents $i, j \in I/\{k\}$ such that $W_i + R_i(g) > 0 > W_j + R_j(g)$. Moreover, by the constrained efficiency of g, there exists no contract b_{ij} such that $i, j \in I/\{k\}$ and $g + b_{ij} \succ_h g$ for each $h \in \{i, j\}$. Furthermore, by the constrained efficiency of g, there exists no contract b_{ik} such that $R_k(g + b_{ik}) \in [0, c]$ and $g + b_{ij} \succ_h g$ for each $h \in \{i, k\}$.

It is left to verify that there is no deviation b_{ik} such that $R_k(g + b_{ik}) \notin [0, c]$, $g + b_{ik} \succ_i g$, and $g + b_{ik} \succ_k g$, that is, deviation that incentivizes k to manipulate the signal. Since $R_i(g) = g_i(2) - g_i(1) \leq 0$ for each $i \in I/\{k\}$ we can focus on deviations b_{ik} that incentivize k to manipulate the signal from 2 to 1. Suppose that there exists such a contract b_{ik}. By assumption, $g + b_{ik} \succ_i g$ and $g + b_{ik} \succ_k g$. Since $R_j(g) \leq 0$ for each $I/\{k, i\}$, $g + b_{ik} \succeq_j g$ for each $j \in I/\{k, i\}$. Since g is constrained-efficient, and $g + b_{ik} \succeq_h g$ for each $h \in I/\{k, i\}$, $g + b_{ik}$ is IR and not degenerate. However, $R_k(g + b_{ik}) \notin [0, c]$, and so the signal is independent of the state, which is in contradiction to IR of g. ■

Corollary 3 *Each constrained-efficient contract is weakly stable.*

When there is only one agent $k \in I$ who is exposed to a shock, a constrained-efficient contract g must be such that $sign(R_k(g)) \neq sign(R_i(g))$ for all $i \in I/\{k\}$. This fact implies that we do not need to worry about the addition of contracts that violate k's incentive constraint. Since only k has the ability to manipulate the signal, no pair of agents $i, j \in I/\{k\}$ can write a contract that incentivizes one of the contracting parties to manipulate the signal. It follows that we do not need to worry about the addition of contracts that impose an externality on third parties.

An additional interesting case is the one in which other agents also have the ability to manipulate the signal. For example, if y is an appraisal, then the insurers may also have the ability to bribe the appraiser. Under the assumption that agent k can also manipulate the signal, the analysis is very similar to the case of $n = |M| = 3$, which will be analyzed in Section 5. Roughly speaking, it must be that in every constrained-efficient contract g, $\sum_{i \in I/\{k\}} |R_i(g)| \leq c$. Therefore, there exists no side-contract b_{ij} between $i, j \in I/\{k\}$ such that $g + b_{ij} \succ g$ for both i and j. In this case, every constrained-efficient contract is pairwise stable. If $k \notin M$, then the stability of constrained-efficient contracts depends on the size of the shock and on the number of agents who can manipulate the signal.

4.3 A joint venture

Each agent $i \in I$ owns an equal share of a venture that is worth nh dollars in state 1 and nl dollars in state 2, $l < h$. Some of the agents are risk-averse while the others are risk-neutral. Denote the set of risk-averse (risk-neutral) agents by L (E). The venture's actual performance is not verifiable. However, the agents can insure each other via contracts contingent on $y \in \{1, 2\}$, which is a variable in the venture's financial reports.

We interpret $M \subseteq I$ as the venture's board of directors (agents who are on the board are assumed to have access to the financial reports). It is assumed that $y = \theta$, unless there is a member of the board who intervenes. The cost of changing the value of y is $c < h - l$. We assume that the number of risk-neutral agents is smaller than the number of risk-averse agents, $|L| > |E| > 1$. Let us compare different boards in the context of the maximal level of insurance that is attainable for risk-averse agents under this board. By Corollary 2, the only deviations that we need to focus on are side-contracts in which $i \notin M$ pays $m \in M$ to incentivize him to manipulate the signal.

4.3.1 Everyone is on the board

Consider the case in which $I = M$. By Corollary 2, each constrained-efficient contract is weakly stable. Since $c < h - l$ and $|L| > |E|$, it must be that $R_i(g) = -c$ for each $i \in E$ in a constrained-efficient contract g. That is, each risk-neutral agent absorbs c dollars of the risk. The total coverage is $c * |E|$.

4.3.2 Risk-averse agents are on the board

Let us assume that $L = M$. By Corollary 2, we need to worry only about deviations in which a risk-neutral agent $i \in E$ pays a risk-averse agent $m \in L$ in order to incentivize m to manipulate the signal. Agent i's willingness to pay for manipulation that guarantees that $y = 1$ regardless of the state is $p_2(g_i(1) - g_i(2))$. It follows that the level of insurance that $i \in E$ can provide is

$$g_i(1) - g_i(2) \leq \min_{m \in M} \left\{ \frac{c - g_m(1) + g_m(2)}{p_2} \right\} \tag{5}$$

When each $i \in E$ provides coverage of c dollars, (5) is slack. As a result, the total coverage that is attainable is strictly greater then the coverage that is attainable when $M = I$.

Can the agents attain the constrained-efficient level of insurance $c * |L|$? For them to obtain this coverage, each $i \in E$ must be able to provide a coverage of at least $\frac{|L|*c}{|E|}$ dollars. Constraint (5) becomes $\frac{|L|*c}{|E|} \leq \frac{c - g_m(1) + g_m(2)}{p_2}$. It follows that the constrained-efficient level of insurance is attainable in a weakly stable contract if $|L| \leq \frac{2}{p_2}|E|$. Note that although the constrained-efficient level of insurance may not be attainable when $M \neq I$, the total insurance provided is strictly greater than in the case of $M = I$. This follows from the fact that the set of constrained-efficient contracts is changed when the board is changed.

4.3.3 Risk-neutral agents are on the board

Let us assume that $M = E$. Since, by assumption $|E| < |L|$ in a constrained-efficient contract, each $i \in E$ provides a coverage of c dollars. As in the previous subsection, we can use Corollary 2 to obtain that the only deviations that can undermine the weak stability of a contract are those that have one agent $i \in L$ who pays an agent $m \in E$ in order to incentivize m to manipulate the signal. Since $|E| < |L|$, i's willingness to pay in order to guarantee that the signal is realized to be 2 does not impose a constraint on the ability to share risk. It follows that the constrained-efficient level of insurance is attainable in a weakly stable contract. Note that the coverage that can be provided by risk-neutral agents is identical to the coverage that they can provide when $M = I$.

4.3.4 A mixture

Let us assume that $M \cap L$, $(I/M) \cap L$, $(I/M) \cap E$, and $M \cap E$ are not empty. That is, there are both risk-averse and risk-neutral agents on and off the board. In a constrained-efficient contract g, each $i \in (I/M) \cap L$ is fully insured and $g_i(2) - g_i(1) = c$ for each $j \in M \cap L$. This is because $(I/M) \cap E \neq \emptyset$. Can this insurance coverage be attained via a weakly stable contract? Note that in a constrained-efficient contract there is at least one agent $i \in L$ on the board who is indifferent to manipulating the signal from 1 to 2, and at least one agent $j \in L$ off the board such that $g_j(2) > g_j(1)$. Such a contract is not weakly stable because j can offer i a contract in which j pays i an arbitrarily small payment conditional on $y = 2$. This contract makes both i and j better off. By Corollary 2, such a contract is not rejected by i.

The mixture of agents on and off the board has three effects. First, the constrained-efficient coverage increases (w.r.t. the case of $L = M$) because some risk-averse agents are now off the board and so they can receive more than c dollars of coverage. Second, in a weakly stable contract we must take into account deviations that include two members of L. This implies that the constrained-efficient level of coverage is not attainable. As we have seen in Application 1, the effect on the amount of coverage that the members of L can receive is ambiguous. Finally, the coverage that the members of E can provide is lower when some of them are on the board.

5 Discussion and extensions

In this section we relax some of our modeling assumptions, namely, symmetry, non-triviality, and richness. We analyze the case of $n = 3$ and study a benchmark model in which risk-sharing is centralized. In Appendix B we extend the model to include multilateral side-contracts.

5.1 Symmetry

Let us relax the assumption that the cost of manipulation is identical for different agents and different signals. Suppose that each $i \in M$ can change the signal's realization from y to y' by paying a cost of $c_i(y \to y')$. What is the effect on the results obtained in the analysis section? Proposition 3 does not rely on symmetry at all. Propositions 1, 2, and 4 rely on the non-triviality of the economy. To obtain Propositions 1, 2, and 4, one should replace $c < W_m$ and $c < -W_{m'}$ in the non-triviality assumption with $c_m(2 \to 1) < W_m$ and $c_{m'}(1 \to 2) < -W_{m'}$ for some $m, m' \in M$. Proposition 5 is more subtle.

First, recall that Proposition 5 follows directly from Lemma 3. The assumption that $c_i(y \to y') = c_i(y' \to y)$ does not play a significant role in the proof of Lemma 3. In fact, this assumption can be completely relaxed. However, the assumption that $c_i(y \to y') = c_j(y \to y')$ for each $i, j \in I$ is used. To see that, recall that the final argument of Lemma 3 was that in order to persuade m' to agree to the deviation $b_{m'm}$, after which he is supposed to manipulate the signal, m must pay m' more than c dollars in the relevant realization. This contradicted the assumption that the offer is beneficial for m, who can manipulate the signal himself by paying c in just one of the realizations. If the difference between $c_m(y \to y')$ and $c_{m'}(y \to y')$ is sufficiently large, it may still be beneficial for m to pay m' more than $c_{m'}(y \to y')$ so that m' will manipulate the signal. We demonstrate this argument with the following example.

Example 4 *The agents' endowments are summarized by the following table:*

agent	1	2	3	4
$w_i(1)$	10	20	100	200
$w_i(2)$	20	10	200	100

Let $p_1 = 0.5$ *and* $u_i(x) = \log(x)$ *for all* $i \in I$. *Let* $c_1(2 \to 1) = 5 = c_2(1 \to 2)$, $c_3(2 \to 1) = 95 = c_4(1 \to 2)$. *In each constrained-efficient contract* g, *all of the agents have binding manipulation constraints. That is,* $R_1(g) = -5$, $R_2(g) = 5$, $R_3(g) = -50$, *and* $R_4(g) = 50$.
Consider agent 4. *Given a constrained-efficient contract, his willingness to pay in order to guarantee that* $y = 2$ *is always greater than* 5. *If agent* 4 *makes an offer* b_{24}, *such that* $b_{24}(2) = 5 + 2\varepsilon$, $b_{24}(1) = 5 + \varepsilon > 5$, *agent* 2 *will never reject it (no matter what his beliefs are). This offer violates agent* 2*'s incentive constraint. Therefore,* g *is not weakly stable.*

Although symmetry (between different agents) is used in the proof of Proposition 5, the result can be obtained with some degree of asymmetry. This follows from the fact that in each constrained-efficient contract g, agent i's willingness to pay for manipulation to $y = 1$ ($y = 2$) is strictly lower than $c_i(2 \to 1)$ ($c_i(1 \to 2)$).

5.2 Non-triviality relaxed: An example

An economy is said to satisfy non-triviality if c is sufficiently low, and, in particular, if $c < W_i, -W_j$ for some $i, j \in M$. This assumption guarantees that in a constrained-efficient contract there is at least one agent who is indifferent between manipulating the signal or not and that, by Claim 2, constrained-efficient contracts are not Pareto-efficient. But what if the possibility of manipulating the signal ex-post does not restrict the agents? In this case, it turns out that the amplification of the moral hazard problem, generated by the ability to write side-contracts, may still result in constrained risk-sharing. To see this consider the following example.

Example 5 *Let* $I = \{1, 2, 3, 4\}$, $M = \{1, 3\}$, *and* $W_1 = W_2 = -W_3 = -W_4 = c$. *Since there is aggregate certainty, a Pareto-efficient contract* g *eliminates the agents' initial exposure such that* $W_i + R_i(g) = 0$ *for each* $i \in I$. *Note that* g *is also constrained-efficient since the non-triviality condition fails in this economy. However,* g *is not weakly stable. To see this note that* $R_3(g) = c$ *and* $R_4(g) > 0$. *Suppose agent* 4 *initiates a deviation* b_{34} *such that* $b_{34}(2) > b_{34}(1) > 0$. *If* $b_{34}(2)$ *is sufficiently low,* $g + b_{34} \succ_4 g$. *By Lemma 2, agent* 3 *does not have a consistent belief that blocks it.*

5.3 An aggregate shock

In most of the analysis section we assumed that the economy is diverse and that Pareto-efficient risk-sharing is precluded by the agents' ability to manipulate the signal. Purely aggregate shocks were ruled out by our assumption that there are two agents i, j such that $W_i > 0 > W_j$. Without loss of generality, we now consider the case in which all agents are (weakly) better off in state 2. That is, $w_j(1) \leq w_j(2)$ for each $j \in I$. Corollary 2 holds in this case. Therefore, Propositions 3 and 5 hold.

The paper's negative results (Propositions 2 and 4) utilize the characterization of constrained-efficient contracts that is obtained in Proposition 1 and that relies on both richness and non-triviality. In particular, they build on the fact that, when richness and non-triviality are satisfied, a constrained-efficient contract must result in two agents $i \notin M$ and $m \in M$ such that m is indifferent to manipulating the signal and $sign(R_m(g)) = sign(R_i(g))$. Application 1 suggests an alternative assumption under which constrained-efficient contracts have this property when the shock is aggregate.

Let us partition I into two disjoint sets, L and E, such that each $i \in E$ is unaffected by the shock ($w_i(1) = w_i(2)$ for each $i \in E$) and cannot manipulate the signal. Suppose that $c < w_m(2) - w_m(1)$ for some $m \in M$ and that $w_i(2) > w_i(1)$ for some $i \in L/M$.

If the cardinality of E is sufficiently large w.r.t. the cardinality of L (or some of the agents in E are risk-neutral), then a constrained-efficient contract must result in two agents $i \notin M$ and $m \in M$ such that m is indifferent to manipulating the signal and $sign(R_m(g)) = sign(R_i(g))$. This guarantees Propositions 2 and 4.

5.4 The case of $n = 3$

First, we show that the result obtained in Proposition 5 holds.

Claim 4 *Let $I = M = \{1, 2, 3\}$. Each constrained-efficient contract is weakly stable.*

To complete the analysis, assume that $|M| = 2$. We now present an example in which there exists no contract that is both constrained-efficient and weakly stable.

Example 6 *Let $I = \{1, 2, 3\}$, $M = \{2, 3\}$, $W_1 = 200, W_2 = -50$, $W_3 = \omega$, where $\omega \in [-100, 0]$. Let $c = 50$ and $u_i(x) = -\exp(-\alpha x)$ for all $i \in I$. Since for each agent i the marginal rate of substitution between consumption in both states depends only on $W_i + R_i(g)$, in every constrained-efficient contract g, it must be that $R_2(g) = R_3(g) = 50$. That is, both agent 2 and agent 3 are indifferent to manipulating the signal or not. We now show that a constrained-efficient contract g is weakly stable only if $\omega = 50$.*

Consider the following deviation b_{ij}, where $i \in \{2, 3\}$, $j \in \{2, 3\} / \{i\}$, and $b_{ij}(2) > b_{ij}(1) > 0$. This contract is signed with the intention that i will manipulate the signal from 1 to 2. Suppose that b_{ij} is offered by j. This deviation is beneficial for j if and only if it violates i's incentive constraint and

$$b_{ij}(2) < x = \frac{1}{\alpha} \log \left[\frac{1 + \frac{(1-p_1)}{p_1} \exp(-\alpha(W_j + R_j(g)))}{\exp(-\alpha R_j(g)) + \frac{(1-p_1)}{p_1} \exp(-\alpha(W_j + R_j(g)))} \right]$$

Suppose that $b_{ij}(2) < x$. We now consider the consistent beliefs that agent i may hold. We will show that if $\omega \neq 50$, then there is a contract b_{ij} that can be offered by i or j, and cannot be blocked by a consistent belief.

Since $b_{ij}(2) > b_{ij}(1) > 0$, a belief $\beta_i(b_{ij}, g)$ such that i is not the one who is supposed to manipulate the signal under $g + \beta_i(b_{ij}, g) + b_{ij}$ cannot be consistent. Therefore, b_{ij} cannot be rejected based on such a belief. It follows that every consistent belief is such that $y(g + \beta_i(b_{ij}, g) + b_{ij}, \theta) = 2$ for each $\theta \in \{1, 2\}$. Let us study the signal's *realizations under* $g + \beta_i(b_{ij}, g)$*. If* $\beta_i(b_{ij}, g)$ *is such that* $y(g + \beta_i(b_{ij}, g), 1) = 1$*, then* j *cannot be worse off agreeing to take part in* b_{ij}*. This is because he does not incur any cost in state* 2 *and he is strictly better off in state* 1*. Therefore, this belief does not allow agent* i *to reject agent* j*'s offer.*

We now complete the analysis by considering an arbitrary consistent belief $\beta_i(b_{ij}, g)$ *such that* $y(g + \beta_i(b_{ij}, g), \theta) = 2$ *for each* $\theta \in \{1, 2\}$*. Agent* i *is better off accepting* b_{ij} *if and only if:*

$$\begin{aligned} & p_1 u_i(w_i(1) + g_i(2)) + p_2 u_i(w_i(2) + g_i(2)) \\ < \; & p_1 u_i(w_i(1) + g_i(2) - c + b_{ij}(2)) + p_2 u_i(w_i(2) + g_i(2) + b_{ij}(2)) \end{aligned}$$

Plugging in the example's parameters, we get that:

$$b_{ij}(2) > \frac{1}{\alpha} \log \left[\frac{\exp(\alpha c) + \frac{1-p_1}{p_1} \exp(-\alpha W_i)}{1 + \frac{1-p_1}{p_1} \exp(-\alpha W_i)} \right]$$

We can find $b_{ij}(2)$ *that satisfies the two conditions if and only if* $W_j > W_i$*. Therefore, these two deviations (one deviation initiated by* i*, the other by* j*) are blocked by a consistent belief only if* $\omega = 50$*. Otherwise, either agent* 2 *or agent* 3 *can make an offer that his counter-party will not reject.*

5.5 Centralized trade

This subsection studies an economy in which, given a contract g, agents who want to write additional contracts must give up on all of their trade relations in g. We have in mind a story in which trade is organized by a planner. A group of agents who want to appeal on the planner's prescribed contract must leave the economy and use the resources of its members. We show that in this case, constrained-efficient insurance is attainable. That is, there exists a constrained-efficient multilateral risk-sharing contract such that no coalition of agents (of any size) has an incentive to deviate from it using the resources of its members (and only these resources). Formally:

Definition 7 *We say that a contract* g *belongs to the core if there exists no other contract* g^K *such that* $g^K \succeq_i g$ *for each* $i \in K$ *(with at least one strong preference).*

Proposition 7 *The core is non-empty.*

Proof. This proof is based on a result obtained by Scarf (1967): a balanced n-person game has a non-empty core. Let T be a collection of subsets of I. We say that T is *balanced* if it is possible to find non-negative weights δ_S for any subset $S \in T$ such that $\sum_{S \in T | i \in S} \delta_S = 1$ for each $i \in I$. For any subset of agents $S \subseteq I$ and contract g^S, define $v_i\left(g^S\right) := p_1 u_i\left(w_i(1) + g_i^S(1)\right) + p_2 u_i\left(w_i(2) + g_i^S(2)\right)$. We say that a profile $(v_i)_{i \in S}$ is attainable for the members of $S \subseteq I$ if there exists an IR contract g^S such that $v_i\left(g^S\right) \geq v_i$ for all $i \in S$. A game is said to be balanced if, for each balanced collection T (of subsets of I) and each profile $(v_i)_{i \in I}$ such that for each $S \in T$, $(v_i)_{i \in S}$ is attainable for the members of S, the profile $(v_i)_{i \in I}$ is attainable for the members of I. We now show that our game is balanced.

Suppose that T is a balanced collection of subsets of I. Consider an arbitrary vector $(v_i)_{i \in I}$ such that $(v_i)_{i \in S}$ is attainable for the members of S, for all $S \in T$. We now show that there exists a contract g such that $v_i(g) \geq v_i$ for each $i \in I$. Let $g_i(y) = \sum_{S \in T | i \in S} \delta_S g_i^S(y)$ for each $i \in I$ and $y \in \{1, 2\}$. Since T is balanced, $g_i(\theta) + w_i(\theta)$ is a convex combination of $\left(g_i^S(\theta) + w_i(\theta)\right)_{S \in T}$. It follows that, for each $i \in I$:

$$\begin{aligned} & v_i(g) = p_1 u_i\left(w_i(1) + g_i(1)\right) + p_2 u_i\left(w_i(2) + g_i(2)\right) \\ \geq \; & \min_{S \in T}\left\{p_1 u_i\left(w_i(1) + g_i^S(1)\right) + p_2 u_i\left(w_i(2) + g_i^S(2)\right)\right\} \geq v_i \end{aligned}$$

We need to verify that $\sum_{i \in I} g_i(y) = 0$ for all $y \in \{1, 2\}$ in order to make sure that g is a contract:

$$\sum_{i \in I} g_i(y) = \sum_{i \in I} \sum_{S \ni i} \delta_S g_i^S(y) = \sum_{S \in T} \delta_S \sum_{i \in S} g_i^S(y) = 0$$

The last thing that we need to check is that g is IC. To see this, note that for each $S \in T$, g^S is IR. Therefore, $\left|R_i\left(g^S\right)\right| \leq c$. Recall that $R_i(g)$ is a convex combination of $\left(R_i\left(g^S\right)\right)_{S \in T}$ and therefore must also lie in $[-c, c]$. ■

The proposition shows that when the agents must abandon their trade relations in order to change the contracts that they have signed, constrained-efficient contracts are stable. This is analogous to exclusive dealership contracts. These contracts are more stable than the contracts that we have studied throughout the paper and, therefore, they allow for better risk-sharing. However, exclusive dealership contracts are more costly to implement since they are more complicated and require costly monitoring.

6 Concluding remarks

In this paper we have studied multilateral risk-sharing using contracts that are contingent on manipulable variables. It was shown that the moral hazard problem that is generated by the manipulability of the contractible variable is significantly enhanced when the agents can write bilateral side-contracts without withdrawing from the multilateral risk-sharing agreement. The reason for this is that these side-contracts can be used to incentivize one of the contracting parties to manipulate the contractible variable.

The main contributions of the paper are as follows. First, we incorporate the idea of manipulation into multilateral risk-sharing. Second, our substantive results establish that when it is possible to manipulate the contractible variable, risk-sharing is highly constrained by the agents' ability to write side-contracts. Third, we contribute to the network formation literature by analyzing a network of contracts with a new externality that results from the ability to manipulate the signal. Finally, at the methodological level, we introduce a coarsening of pairwise stability in the tradition of cooperative game theory by incorporating insights from the Nash equilibrium refinements literature.

Throughout the paper we studied the stability and efficiency properties of one multilateral contract. We ignored the structure of the multilateral contract and assumed that the side-contracts are bilateral. This raises two questions. First, is the restriction to bilateral contracts a constraint on multilateral risk-sharing? A result obtained by Rader (1968) allows us to answer this question by suggesting one possible intuitive structure for constrained-efficient contracts: a collection of IR bilateral contracts. However, Rader's result only implies that there exists a constrained-efficient contract that can be decomposed this way. In general, not all constrained-efficient multilateral contracts can be decomposed into IR bilateral contracts. The second interesting question is what would change in our results if we allowed for multilateral side-contracts. We answer this question in Appendix B.

A key ingredient in the model is the fact that contracts are contingent on a signal about the state rather than contingent on the state itself. When contracts are state-contingent, agents cannot avoid being exposed to externalities by reducing their coverage. This is due to the fact that manipulation may have an effect on the agents' endowments. Another important difference between state-contingent contracts and contracts that are contingent on a state-dependent signal is that under the former, there are cases in which an agent may be better off manipulating the signal himself rather than having another agent doing the manipulating and paying the cost. In these instances, constrained-efficient contracts are pairwise stable. An explicit analysis is left for future research.

Finally, we comment on the agents' motivation to trade. The motivation for contracting in this paper is risk-sharing. However, besides Proposition 1, the model does not make particular use of this motivation. A similar analysis (left for future research) could be made for other trading motivations, such as state-dependent utilities or differences in the agents' prior beliefs. The main difference between risk-sharing and non-common prior beliefs as a motivation for contracting is that risk-sharing (speculative trade) contracts are used to reduce (increase) the agents' exposure to the state of nature.

Related Literature

This work is related to the risk-sharing networks literature. Bramoulle and Kranton (2007a, 2007b) study risk-sharing network formation models in which agents commit to sharing monetary holdings equally with linked partners. In these models, the agents trade off between costly link formation and better risk-sharing. Bloch, Genicot, and Ray (2008) and Ambrus, Mobius, and Szeidl (2014) consider moral hazard in risk-sharing networks. In these models, ex-post, an agent who is expected to make a transfer to a network neighbor may prefer to deviate and withhold payment. An agent who deviates loses some of his risk-sharing links. Bloch, Genicot, and Ray (2008) take the risk-sharing agreements as given and characterize stable networks while Ambrus, Mobius, and Szeidl (2014) take the network as given and study the degree and structure of risk-sharing.

The model presented in this work is in the spirit of the strategic network formation literature. Jackson and Wolinsky (1996) study a model in which agents' payoffs depend on links formed by them and their counter-parties. Jackson and Watts (2002) study the dynamic formation of networks and show that, in principle, pairwise-stable networks need not exist. Erol and Vohra (2015) study a model of systemic risk in which agents form binary links, then learn a realization of some shock, and, based on this shock, each agent has to decide on a binary action. In their model, the payoff of an agent depends on the action he takes in the third stage, the actions of his neighbors, and the realized shocks. Roketskiy (2015) studies strategic network formation in the context of collaboration in winner-takes-all tournaments. He uses the farsighted stable set as a solution concept and shows that the set of pairwise-stable networks and the farsighted stable set may be disjoint.

Pomatto (2015) applies forward-induction reasoning to an incomplete information matching problem using a non-cooperative approach. He considers given allocations and models a deviation game in which agents negotiate via take-it-or-leave-it offers. An allocation is stable if in its deviation game agents abstain from making offers, expect no offer to be made by other agents, and in case an offer is made they interpret it with the highest degree of strategic sophistication that can be ascribed to its proposer. Weak stability incorporates forward-induction considerations of this kind into a stability notion in the tradition of cooperative game theory.

Weak stability is related to the Nash equilibrium refinements literature (see, e.g., Kohlberg and Martens, 1986). A detailed discussion about its analogy to Cho and Kreps' (1987) intuitive criterion is to be found in Section 4.3. Another related concept is Riley's (1979) reactive equilibrium, in which agents react to observed deviations by additional deviations. In the context of cooperative game theory, weak stability is related to the counter-objections literature (see, e.g., Aumann and Maschler, 1961, and Mas-Colell, 1989) that is used to overcome cases in which the core of an n-person cooperative game is empty. Counter-objections are used to reduce the set of possible deviations, as a deviation that has a counter-objection is not allowed.

Eliaz and Spiegler (2007, 2008, 2009) take a mechanism-design approach to problems in which agents are motivated to bet on the state of nature due to differences in their prior beliefs. A major obstacle to the analysis of speculative trade is that risk-neutral traders with different prior beliefs are willing to take infinite bets. Eliaz and Spiegler assume that the agents can manipulate the contractible variable by incurring some cost. This assumption creates incentive constraints that restrict the betting stakes.

Duffie and Stein (2015) study economic benchmarks such as the inter-bank offered rates. They explain the importance of these benchmarks in reducing market frictions and how trade is agglomerated around these benchmarks. Duffie and Stein explain how manipulation occurs in practice in these markets, and illustrate how benchmark definitions and fixing methods can mitigate manipulation.

Greenwald and Stiglitz (1986) establish that when there are distortions (for example, incomplete markets or imperfect information) the first welfare theorem does not apply: competitive economies are not constrained-efficient. The inefficiency results obtained in the present paper are in the spirit of Greenwald and Stiglitz's result. However, the two follow from different strategic considerations.

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7 Appendix A: Manipulation games

Throughout the analysis it was assumed that when a contract is not IC, the agent who receives the most coverage (in absolute value), $m \in M$, sets the signal and pays for manipulation when it is necessary (i.e., when $y(g, \theta) \neq \theta$). In this appendix we discuss this assumption and present two manipulation games in which it is obtained as a result of an equilibrium. Later, we present three additional games in which the above-mentioned assumption is not supported by an equilibrium. However, we show that our main result (Proposition 4) holds when we replace the aforementioned assumption with each of the the latter manipulation games.

We mention that the agents play *two* manipulation games, one in each state. The primitives of each manipulation game Γ include a set of players M, a state θ, endowments $(w_i(\theta))_{i\in M}$, and a contract g. In each game, the set of actions available to each agent includes at least one action that is costless and is interpreted as not trying to manipulate the signal. We omit the set of players, actions, and endowments from the description of a manipulation game $\Gamma(\theta, g)$ and denote the signal that results from the manipulation game by[7] $y(\Gamma(\theta, g))$.

Throughout this appendix we focus on pure strategy equilibria. This is for two reasons. First, mixed strategy equilibria of the games we present have the unappealing property that the greater one's incentive to manipulate the signal is, the lower the probability that he actually manipulates it. This unattractive property of mixed-strategy equilibria in concession games and war of attrition games is well known. Second, in many of the games described in this section as well as their variations, mixed strategy equilibria are not robust in the sense that there always exists a group of agents who can improve their payoff by coordinating using a jointly controlled lottery (Aumann, Maschler, and Stearns, 1968) before the start of the game.

Before we start analyzing the different manipulation games, let us state the following conventions and notation. Let $D(g) := \{i \in PM(g) \,|R_i(g) < 0\}$, $U(g) := \{i \in PM(g) \,|R_i(g) > 0\}$. That is, $D(g)$ ($U(g)$) is the set of manipulators who are incentivized to manipulate the signal from 2 (1) to 1 (2). We assume that if an agent is indifferent between using an action that is interpreted as manipulation of the signal or not, then he does not manipulate the signal. In the three manipulation games in which our assumption is not obtained as a result of an equilibrium (manipulation games 3, 4, and 5), we check the robustness of the paper's main result (Proposition 4). We do so by showing that Lemma 2 holds, as this is the only part of the proposition in which the assumption that we relax in the appendix is used. The lemma demonstrated one deviation that is not blocked by a consistent belief. Without loss of generality, in the appendix, we study one deviation b_{ij}, initiated by $j \notin M$, such that $b_{ij}(2) > b_{ij}(1) > 0$, $i \in M$, and $g + b_{ij} \succ_j g$. Since $g + b_{ij} \succ_j g$, it follows that $R_i(g + b_{ij}) > c$. We consider an arbitrary belief of the offer's receiver $\beta_i(b_{ij}, g)$ and show that either $\beta_i(b_{ij}, g)$ is inconsistent or that it does not block b_{ij}. If this deviation is not blocked, agent i is assumed to accept it. In this case, it is an equilibrium for him to manipulate the signal such that the resulting signal is 2 regardless of the state of nature.

[7] We allow the result of a manipulation game to be a distribution of signals.

7.1 Game 1: Repeated opportunities to manipulate the signal

The manipulation game Γ_1 is sequential. In each period one agent $i \in M$ is given an opportunuity to manipulate the signal. Agent i is informed about the state of nature and the current value of the signal. He decides whether to manipulate the signal or not. In case i decides to manipulate the signal, he pays c, and the signal's value changes. In each period there is an exogenous probability of $1-\delta$ that the game ends immediately. The outcome of the game is the signal at its end. Agent i's payoff is $u_i\left(w_i\left(\theta\right)+g_i\left(y'\right)-tc\right)$, where y' is the signal that results from the game and t is the number of times i has manipulated the signal.

Our interpretation of Γ_1 is of a situation in which the agents have random access to a functionary/appraiser that he can bribe. We now present a selection between the possible equilibria of Γ_1. This selection is based on the idea that the agent who has the most to gain is perceived by others to be more eager to manipulate the signal.

Equilibrium that supports our assumption

If g is IC, then it is an equilibrium for each $i \in M$ never to manipulate the signal. If g is not IC, then there exists a stopping probability $\delta < 1$ for which the following profile of strategies is a sub game perfect Nash equilibrium. The agent who receives the most coverage (in absolute value) always manipulates the signal to his preferred realization when it is his turn to play. The other agents never try to manipulate the signal. Note that for any contract g, δ must be sufficiently close to 1 in order for the aforementioned profile to be a subgame perfect Nash equilibrium of Γ_1. If we do not allow the agents to trade such that their wealth falls below some threshold $z \leq 0$ (or put any other restriction on the size of the contracts' stakes), then there is a probability δ sufficiently close to 1 that covers all admissible contracts.

7.2 Game 2: Second-price auction

The members of M play a second-price auction (with a symmetric tie-breaking rule) whose winner pays the second-highest bid and gets the right to set the signal. He can pay additional c dollars and manipulate the signal or keep the signal's value at no cost.

Here we interpret c as a combination between the cost of manipulation. The agents can be interpreted as a regulator that is lobbied or a functionary that is bribed. In our selected equilibrium the functionary receives no bribe. This as an implicit assumption that the regulator/functionary is not strategic.

An equilibrium that supports our assumption

The (lowest labeled) agent with the most coverage (in absolute value) $i \in PM(g)$ submits a bid $b_i = |R_i(g)|$ and all of the other agents submit 0. If $R_i(g) > c$ ($R_i(g) < -c$), agent i pays c in the case where $\theta = 1$ ($\theta = 2$). Note that bidding 0 is not dominated by bidding one's coverage since manipulation is a public good (it may be better to lose the auction and avoid paying for manipulation).

7.3 Game 3: One opportunity to manipulate the signal

In the manipulation game Γ_3, nature randomly draws an order according to which the agents are called to play. Each agent plays once. An agent who is called to play learns the state and the current value of the signal. He chooses to manipulate the signal and pay c or not. He does not know how many players have played or how many times the signal was manipulated before (but when the signal does not match the state, he can infer that there has been at least one manipulation). The outcome of the game is the signal y at its end. Player i's payoff is $u_i(w_i(\theta) + g_i(y) - c)$ ($u_i(w_i(\theta) + g_i(y))$) if he manipulates (does not manipulate) the signal.

Existence of pure strategy equilibria

The existence of a pure strategy equilibrium is not straightforward, and to the proof of its existence we now turn. Assume without loss of generality that $\theta = 1$. If $D(g)$ is empty, then there exists a pure strategy equilibrium in which no agent manipulates the signal to $y = 1$. If $U(g)$ is empty, then there exists a pure strategy equilibrium in which no one manipulates the signal. We now assume that both $U(g)$ and $D(g)$ are not empty.

Fix the strategies of the members of $U(g)$ such that one agent $i \in U(g)$ manipulates the signal when he observes $y = 1$ and the other agents never manipulate it. Then, partition $D(g)$ into two disjoint sets (where one of the sets is allowed to be empty) such that all members in $D_0^+(g)$ ($D_0^-(g)$) manipulate (do not manipulate) the signal when they observe $y = 2$ and this is a best reply to the other agents' actions (members of $D(g)$ do not manipulate the signal when they observe $y = 1$). There exist two such sets since the probability of being pivotal (agents want to manipulate the signal only if they are pivotal) is decreasing in the number of agents who manipulate the signal. Note that these strategies are also best replies to a profile of strategies in which all members of $U(g)$ do not manipulate the signal under the belief that if an unexpected manipulation has happened (that is, if the signal's value is 2 although there were no expected manipulations), a member of $U(g)$ caused it (this belief is the only one that survives the intuitive criterion).

Fix the aforementioned actions of the members of $D_0^+(g)$ and $D_0^-(g)$ and consider $U(g)$. Create the two disjoint sets $U_0^+(g)$ and $U_0^-(g)$ in a similar way. That is, all members of $U_0^+(g)$ ($U_0^-(g)$) manipulate (do not manipulate) the signal when they observe $y=1$. If $\left|U_0^+(g)\right| \in \{0,1\}$, then we have found an equilibrium. If $\left|U_0^+(g)\right| > 1$, then fix the strategies of the members of $U_0^+(g)$ and $U_0^-(g)$ and consider the members of $D(g)$ again. Partition $D(g)$ into two disjoint sets (where one of the sets is allowed to be empty) such that all members in $D_1^+(g)$ ($D_1^-(g)$) manipulate (do not manipulate) the signal when they observe $y=2$ and this is a best reply to the other agents' actions. It must be that $\left|D_1^+(g)\right| \leq \left|D_0^+(g)\right|$ since the probability of each $i \in D(g)$ being pivotal given $U_0^+(g)$ and $U_0^-(g)$ is smaller than this probability under the assumption that only one member of $U(g)$ manipulates the signal. Similarly, fix $U_1^+(g)$ and $U_1^-(g)$ and observe that $\left|U_1^+(g)\right| \geq \left|U_0^+(g)\right|$. Continue with this algorithm until $\left|U_i^+(g)\right| = \left|U_{i+1}^+(g)\right|$. Note that the algorithm must end since $\left|U_i^+(g)\right| \geq \left|U_{i-1}^+(g)\right|$ for any stage $i>1$ and M is finite. The algorithm ends in an equilibrium since each agent's action is a best response to the other agents' actions.

Robustness of Proposition 4

The game Γ_3 may have multiple equilibria. Before we prove the robustness of Proposition 4, we present a selection criterion that we refer to as *no sunspots*. This criterion will be used to select between the different equilibria of Γ_3. No sunspots implies that when some agent j's payoff is changed, the selection between equilibria in which j does not manipulate the signal is not changed.

Formally, we say that there are no sunspots if the following condition is met for each two contracts g and g' that differ only in an arbitrary agent j's payoff, and for each two profiles of strategies $(s_i)_{i\in M}$ and $(s_i')_{i\in M}$ in which j does not manipulate the signal.

- If $(s_i)_{i\in M}$ and $(s_i')_{i\in M}$ are both equilibria of the manipulation game $\Gamma_3 = (\theta, g)$, $(s_i)_{i\in M}$ is an equilibrium of the manipulation game $\Gamma_3' = (\theta, g')$, and $(s_i)_{i\in M}$ is played in $\Gamma_3 = (\theta, g)$, then $(s_i')_{i\in M}$ is not played in $\Gamma_3' = (\theta, g')$.

We now show that the result obtained in Proposition 4 holds. We need to show that under any consistent belief $\beta_i(g, b_{ij})$ that agent i may hold, the equilibria of $\Gamma_3(1, g+\beta_i(g, b_{ij}))$ and $\Gamma_3(2, g+\beta_i(g, b_{ij}))$ are not changed in a way that makes i worse off if he agrees to b_{ij}. Recall that in the contract b_{ij} agent j pays agent i a strictly positive amount in both realizations of the signal, and that consistency implies that j is not worse off by making this offer to i.

There are four different cases, namely, $U(g + \beta_i(g, b_{ij}))$ can be empty or not, and $D(g + \beta_i(g, b_{ij}))$ can be empty or not. In the case in which both $U(g + \beta_i(g, b_{ij}))$ and $D(g + \beta_i(g, b_{ij}))$ are empty, it is straightforward to see that i is better off agreeing to b_{ij}. We now analyze the other three cases.

Case 1: Let us assume that $U(g + \beta_i(g, b_{ij}))$ is not empty and $D(g + \beta_i(g, b_{ij}))$ is empty. It must be the case that $y(\Gamma_3(\theta, g + \beta_i(b_{ij}, g))) = y(\Gamma_3(\theta', g + \beta_i(b_{ij}, g) + b_{ij})) = 2$ for each $\theta, \theta' \in \{1, 2\}$. Therefore, $\beta_i(b_{ij}, g)$ is inconsistent.

Case 2: Let us assume that $D(g + \beta_i(g, b_{ij}))$ is not empty and $U(g + \beta_i(g, b_{ij}))$ is empty. Then, $y(\Gamma_3(\theta, g + \beta_i(b_{ij}, g))) = 1$ for each $\theta \in \{1, 2\}$. Agent i can guarantee himself a payoff of at least $u_i(w_i(\theta) + g_i(1) + b_{ij}(1)) > u_i(w_i(\theta) + g_i(1))$ regardless of the other agents' actions. It follows that i cannot be worse off agreeing to b_{ij}.

Case 3: Suppose that $U(g + \beta_i(g, b_{ij}))$ and $D(g + \beta_i(g, b_{ij}))$ are both not empty. We start with the game $\Gamma_3(1, g + \beta_i(g, b_{ij}) + b_{ij})$. We can set $R_i(g) - c > 0$ to be arbitrarily small, such that i manipulates the signal from 1 to 2 only if he is the only one to manipulate the signal (i.e., he is pivotal with probability 1). If there exists such an equilibrium, then each agent $k \in D(g + \beta_i(g, b_{ij}))$ who observes that the signal's value is 2 knows that i has already manipulated it, and therefore agent k is pivotal with probability 1. It follows that agent k's unique best response in this case is to manipulate the signal. Since by assumption $D(g + \beta_i(g, b_{ij}))$ is not empty, there exists no such equilibrium in $\Gamma_3(1, g + \beta_i(g, b_{ij}) + b_{ij})$. By no sunspots, the equilibrium played does not change because of the addition of b_{ij} and so $\Pr\{y(\Gamma_3(1, g + \beta_i(g, b_{ij}) + b_{ij})) = \theta\} = \Pr\{y(\Gamma_3(1, g + \beta_i(g, b_{ij}))) = \theta\}$ for each $\theta \in \{1, 2\}$.

Let us consider $\Gamma_3(2, g + \beta_i(g, b_{ij}) + b_{ij})$. By the previous argument, the signals' distribution that results from $\Gamma_3(2, g + \beta_i(g, b_{ij}) + b_{ij})$ is different from the one that results from $\Gamma_3(2, g + \beta_i(g, b_{ij}))$. Otherwise, $\beta_i(b_{ij}, g)$ is not consistent. Recall that we set $R_i(g) - c > 0$ to be small such that i manipulates the signal only if he is pivotal with probability 1, that is, if there is no $k \in U(g + \beta_i(g, b_{ij}))$ that manipulates the signal and there is at most one agent in $D(g + \beta_i(g, b_{ij}))$ who manipulates it. If there exists no such equilibrium, then by no sunspots, $\beta_i(b_{ij}, g)$ is not consistent.

Assume that there exists an equilibrium of $\Gamma_3(2, g + \beta_i(g, b_{ij}) + b_{ij})$ in which i manipulates the signal, no other member of $U(g + \beta_i(g, b_{ij}))$ manipulates it, and the number of members of $D(g + \beta_i(g, b_{ij}))$ who manipulate it is[8] $z \in \{0, 1\}$. Let us

[8]It is assumed that if $z = 0$, and an agent h observes $y = 1$, he believes that this unexpected manipulation results from the action of some agent k such that $R_k(g + \beta_i(b_{ij}, g)) < 0$. Any other

consider $\Gamma_3(2, g + \beta_i(g, b_{ij}))$ and the number of members of $U(g + \beta_i(g, b_{ij}))$ who manipulate the signal in the equilibrium that is played. Denote this number by v. If $v = 1$, then it must be that z members of $D(g + \beta_i(g, b_{ij}))$ manipulate the signal. This belief is inconsistent since the signals' distribution that results from $\Gamma_3(2, g + \beta_i(g, b_{ij}) + b_{ij})$ is not different from the one that results from $\Gamma_3(2, g + \beta_i(g, b_{ij}))$. Let us assume that $v \neq 1$.

Suppose that $v = 0$. Since $D(g + \beta_i(g, b_{ij}))$ is not empty, exactly one of its members manipulates the signal. This cannot be a part of an equilibrium since $U(g + \beta_i(g, b_{ij}))$ is not empty and each of its members is pivotal with probability 1.

Suppose that $v \geq 2$. It follows that there is an agent $h \in U(g + \beta_i(g, b_{ij})) / \{i\}$ who prefers to manipulate the signal when i is the only one to manipulate it from 1 to 2 and $z \in \{0, 1\}$. This is in contradiction to the existence of an equilibrium of $\Gamma_3(2, g + \beta_i(g, b_{ij}) + b_{ij})$ in which i manipulates the signal.

To conclude, a belief can be consistent only if $U(g + \beta_i(g, b_{ij}))$ is empty. Therefore, i is not worse off agreeing to b_{ij}. It follows that the result obtained in Lemma 2 holds.

7.4 Game 4: Manipulation with an aggressiveness relation

Let Q be a strict linear ordering according to which the agents are ordered. The interpretation of hQk is "h is stronger than k". Each agent decides . He observes the actions of those who played before him and does not know the order of the agents who will play after him. Each agent chooses between the three actions $+$, $-$, and $=$. The first two actions are interpreted as manipulating the signal to 2 and 1, respectively. Their cost is c. The third action is costless and is interpreted as not trying to manipulate the signal. The signal is set to θ unless some of the agents manipulate it. In the latter case, it is set according to the action of the strongest agent who chose to manipulate the signal.

In this game we assume that there is one agent who is stronger than the others. His action defeats any action taken by other agents. Unlike the first two examples, the strength relation is exogenous in this case.

Robustness of Proposition 4

It is easy to see that the signal must be set in favor of the strongest agent in $PM(g)$. Therefore, consistency of $\beta_i(b_{ij}, g)$ implies that i must be the strongest agent in $PM(g + \beta_i(b_{ij}, g) + b_{ij})$. Also, if $PM(g + \beta_i(b_{ij}, g)))$ is not empty, consistency

belief does not survive the intuitive criterion.

of $\beta_i(b_{ij}, g)$ implies that the signal that results from the contract $g + \beta_i(b_{ij}, g)$ is 1 regardless of the state of nature. Therefore, i cannot be worse off agreeing to b_{ij}.

8 Appendix BL Multilateral side-contracts

In the paper we restricted the size of the deviating coalition. We only allowed for deviations by pairs of agents. We now generalize our solution concept and allow for larger deviating coalitions. In addition, we allow the agents' beliefs to consist of multilateral contracts. Formally, an offer is a pair (i, g^K), where i is the proposer of the offer and g^K is a multilateral contract such that $i \in K \subseteq I$. A belief $\gamma_j(i, g^K, g)$ is a collection of other contracts $(g^{K'})_{\substack{i \in K' \\ K' \in 2^{I/\{k\}}}}$ proposed by i. Note that a belief is a function of the current contract g, the proposer i, and the offered contract g^K.

Definition 8 *A belief $\gamma_k(i, g^K, g)$ is group-consistent if $g + g^K + \gamma_k(i, g^K, g) \succ_i g + \gamma_k(i, g^K, g)$.*

Definition 9 *A contract g is group weakly-stable if, for any contract g^K such that $g + g^K \succ_i g$ for some $i \in K$, there exists another agent $j \in K/\{i\}$ and a group-consistent belief $\gamma_j(i, g^K, g)$ such that $g + \gamma_j(i, g^K, g) + g^K \prec_j g + \gamma_j(i, g^K, g)$.*

We now establish that the inefficient risk-sharing problem under group-weak-stability is more severe than under weak stability.

Proposition 8 *If a contract g is not weakly stable, then it is not group-weakly stable.*

Proof. Assume by negation that g is not weakly stable and is group-weakly stable. It follows that there is a deviation b_{ij}, initiated by i, that is not blocked by any consistent belief $\beta_j(b_{ij}, g)$, but is blocked by a group-consistent belief $\gamma_j(i, b_{ij}, g)$. Consider the multilateral contract that sums all of the contracts that are included in $\gamma_j(i, b_{ij}, g)$ and denote it by g'.

Note that consistency and group-consistency impose no restrictions on agent j's beliefs about the considerations of each agent $k \neq i$. Moreover, there are no restrictions on agent j's beliefs about i's considerations w.r.t. contracts that he signs with other agents. In particular, these contracts need not be IR for i or his counter-parties to the contracts. Therefore, any belief that consists of a collection of bilateral contracts (such that i is one side of each contract) that induces g' is consistent. Since contracts are budget-balanced transfers between the agents, there exists such a collection of bilateral contracts. ■

It turns out that some contracts and, in particular, constrained-efficient ones can be weakly stable but not group-weakly stable. The reason for this is that multilateral deviations initiated by members of I/M allow them to split the cost of incentivizing others to manipulate the signal. In the next result we demonstrates this fact. We enlarge the economy and show that the aggregate coverage that can be provided to "non-manipulators" (in an IR group-weakly stable contract) is bounded from above.

We now describe a procedure to enlarge the economy. We imagine the economy to be composed of n types of agents with r agents of each type. For two agents to be of the same type we require that they have the same preferences, the same manipulation cost, and the same initial resources. Given an economy $E = \left((u_i)_{i \in I}, (w_i(\theta))_{\substack{i \in I \\ \theta \in \{1,2\}}}, p_1, c, M, I \right)$, we define the replica economy E_r to be the economy E with r agents of each type.

Proposition 9 *Fix an economy* $E = \left((u_i)_{i \in I}, (w_i(\theta))_{\substack{i \in I \\ \theta \in \{1,2\}}}, p_1, c, M, I \right)$ *such that* $M \neq I$. *There exists a number* $T > 0$ *such that for each economy* E_r, *and each IR group-weakly stable contract* g, $\sum_{i \notin M | R_i(g) > 0} R_i(g) \leq T$ *and* $\sum_{i \notin M | R_i(g) < 0} R_i(g) \geq -T$.

Proof. Let us consider an IR contract g and an agent $i \notin M$. Consider $K \subseteq I$ such that $\{m\} = K \cap M$ and $i \in K$. Suppose i makes an offer g^K such that $g_i^K(1), g_i^K(2) < 0$, $R_m\left(g + g^K\right) > c$, and $g + g^K \succ_i g$. That is, agent i suggests a contract in which he receives a negative payoff in both of the signal's realizations, and $g_m(2) > g_m(1)$. By the arguments made in Lemma 2, agent m has no consistent belief (or group-consistent belief) that blocks this contract. Consider agent $k \in K/\{i, m\}$. Since $g_i^K(1), g_i^K(2) < 0$, group-consistency of $\gamma_k\left(i, g, g^K\right)$ implies that m changes his action because of g^K. That is, either m manipulates the signal from 1 to 2 because of g^K (and according to $g + \gamma_k\left(i, g, g^K\right)$ the signal is realized to be 1 in state 1), or m was supposed to manipulate the signal from 2 to 1 according to $g + \gamma_k\left(i, g, g^K\right)$ and he does not manipulate the signal given the addition of g^K. In both cases, because of g^K, the realization of the signal is changed from 1 to 2 in (at least) one of the states.

Each agent $k \in K/\{i, m\}$ such that $R_k(g) > 0$ is willing to pay a strictly positive amount $x_k(g)$ for this change in m's behavior. Since IR implies that g is non-manipulable, it must be that $|R_m(g)| \leq c$ for all $m \in M$. It follows that $\sum_{k \notin M | R_i(g) > 0} x_k(g) \leq 2c$ in every weakly stable contract g. As a result, there is a number T such that $\sum_{i \notin M | R_i(g) > 0} R_i(g) \leq T$. The proof of the symmetric case is omitted. ■

Corollary 4 *The maximal average coverage* $\frac{\sum_{i \notin M} |R_i(g)|}{|I/M|}$ *that can be attained (via a* *group-weakly stable contract) by agents who are unable to manipulate the signal goes to* 0 *as* r *goes to infinity.*

It turns out that enlarging the size of the deviating coalition does not have an effect on the stability of constrained-efficient contracts when $M = I$. In Proposition 5 it is established that constrained-efficient contracts are weakly stable when $M = I$. Since the proof of the group-weak stability of constrained-efficient contracts when $M = I$ is very similar to the proof of Lemma 3, it is omitted.

9 Appendix C: Proofs

9.1 Proof of Claim 1

If g is not IC, the signal is realized to be $y \in \{1, 2\}$ regardless of θ. Since the signal is independent of the state, any non-degenerate contract g makes at least one of the agents worse off. Since g is not IC, it is non-degenerate.

9.2 Proof of Claim 2

It is well known that in every Pareto-efficient contract g', $W_i + R_i(g') \leq 0$ for each $i \in I$ or $W_i + R_i(g') \geq 0$ for each $i \in I$. Since $c < W_i$, the constrained-efficiency of g implies that $W_i + R_i(g) > 0$. Since $c < -W_j$, the constrained-efficiency of g implies that $W_j + R_j(g) < 0$. It follows that g is not Pareto-efficient.

9.3 Proof of Lemma 3

Let us consider a deviation $b_{m'm}$, initiated by m, such that $g + b_{m'm} \succ_m g$. Suppose that $b_{m'm}(1) < 0$. That is, m' pays m a strictly positive amount if the signal is realized to be 1. We will now find a consistent belief $\beta_{m'}(b_{m'm}, g)$ that blocks this deviation. Since $n > 3$, there exist two agents $l, k \in I/\{m, m'\}$. Let $\beta_{m'}(b_{m'm}, g) = (b_{mk}, b_{ml})$ such that $b_{mk}(1) - b_{mk}(2) = b_{ml}(1) - b_{ml}(2) = d$. One can choose d large enough such that $g_m(1) - g_m(2) + 2d > \max\{c, R_{m'}(g + b_{m'm}), R_k(g) + d, R_l(g) + d\}$. It follows that under $\beta_{m'}(b_{m'm}, g)$, m sets the signal to be 1 in both states. Since $b_{m'm}(1) < 0$, $\beta_{m'}(b_{m'm}, g)$ is consistent and blocks $b_{m'm}$. The case of a deviation $b_{m'm}$ such that $b_{m'm}(2) < 0$ is symmetric and its proof is omitted.

To complete the analysis, we now consider deviations $b_{m'm}$ such that $b_{m'm}(y^*) > b_{m'm}(y') \geq 0$ for $y^* \neq y'$. Without loss of generality, assume that $y^* = 2$. It follows that $R_m(b_{m'm}) = b_{m'm}(1) - b_{m'm}(2) < 0$. Since $|M| \geq 3$, there exists an agent $k \in M/\{m', m\}$. Consider a belief $\beta_{m'}(b_{m'm}, g) = (b_{mk}, b_{ml})$ such that $k \in M/\{m, m'\}$ and $l \in I/\{m, m', k\}$. Fix the two contracts b_{mk}, b_{ml} such that

$$\begin{aligned} -R_m(g + b_{mk} + b_{ml} + b_{m'm}) &> R_k(g + b_{mk}) > \\ -R_m(g + b_{mk} + b_{ml}) &> \max\{R_{m'}(g + b_{m'm}), |R_l(g + b_{ml})|, c\} \end{aligned}$$

The contract $b_{m'm}$ changes the realization of the signal in both states since it changes the identity of the agent who decides on it. That is, $y(g + b_{mk} + b_{ml} + b_{m'm}, \theta) = 1$ and $y(g + b_{mk} + b_{ml}, \theta) = 2$ for each $\theta \in \{1, 2\}$. Note that $|R_m(b_{mk} + b_{ml})|$ can be set to be sufficiently large such that $\beta_{m'}(b_{m'm}, g)$ is consistent. Agent m' is not worse off agreeing to $b_{m'm}$ according to $\beta_{m'}(b_{m'm}, g)$ only if

$$\begin{aligned} & p_1 u_{m'}(w_{m'}(1) + g_{m'}(2)) + p_2 u_{m'}(w_{m'}(2) + g_{m'}(2)) \\ \leq \; & p_1 u_{m'}(w_{m'}(1) + g_{m'}(1) + b_{m'm}(1)) + p_2 u_{m'}(w_{m'}(2) + g_{m'}(1) + b_{m'm}(1)) \end{aligned}$$

This implies that $R_{m'}(g) = g_{m'}(2) - g_{m'}(1) \leq b_{m'm}(1)$. Since g is IR and $g + b_{m'm} \succ_m g$, it must be that m' is supposed to decide on the signal given $g + b_{m'm}$. Therefore, $|R_{m'}(g + b_{m'm})| > c$. Since $b_{m'm}(2) > b_{m'm}(1)$, $R_{m'}(g + b_{m'm}) > c$. It follows that $b_{m'm}(2) - b_{m'm}(1) + R_{m'}(g) > c$. As a result, $b_{m'm}(2) > c$. But this implies that $g + b_{m'm} \prec_m g$.

9.4 Proof of Claim 4

Let g be a constrained-efficient contract. Without loss of generality, assume that $R_1(g) < 0$, $R_2(g), R_3(g) \geq 0$. Since g is constrained-efficient, each contract b_{ij} such that $g + b_{ij} \succ_i g$ and $g + b_{ij} \succ_j g$ must violate one of the counter-parties' incentive constraints. Otherwise, g is Pareto-dominated by $g + b_{ij}$. This implies that the only candidates to undermine the weak stability of g are contracts between agents 2 and 3. Moreover, if there exists a contract b_{23} such that $g + b_{23} \succ_3 g$ and $g + b_{23} \succ_2 g$, then $R_2(g) > 0$ and $R_3(g) > 0$. Furthermore, since $c \geq |R_1(g)| = R_2(g) + R_3(g)$, $R_2(g), R_3(g) < c$. Since agent 2's and agent 3's incentive constraints are not binding, and g is constrained-efficient, the two agents have the same marginal rates of substitution between consumption in the two states ($\frac{u_2'(w_2(1)+g_2(1))}{u_2'(w_2(2)+g_2(2))} = \frac{u_3'(w_3(1)+g_3(1))}{u_3'(w_3(2)+g_3(2))}$).

Assume without loss of generality that there exists a contract b_{23} such that $g + b_{23} \succ_3 g$ and $g + b_{23} \succ_2 g$, that incentivizes agent 3 to manipulate the signal from 1 to 2. Since

$g + b_{23} \succ_3 g$, it must be that

$$\begin{aligned} & p_1 u_3 (w_3 (1) + g_3 (1)) + p_2 u_3 (w_3 (2) + g_3 (2)) \\ < \; & p_1 u_3 (w_3 (1) + g_3 (2) - b_{23} (2) - c) + p_2 u_3 (w_3 (2) + g_3 (2) - b_{23} (2)) \\ \leq \; & p_1 u_3 (w_3 (1) + g_3 (2) - b_{23} (2) + R_1 (g)) + p_2 u_3 (w_3 (2) + g_3 (2) - b_{23} (2)) \\ = \; & p_1 u_3 (w_3 (1) + g_3 (2) - b_{23} (2) - R_2 (g) - R_3 (g)) + p_2 u_3 (w_3 (2) + g_3 (2) - b_{23} (2)) \\ = \; & p_1 u_3 (w_3 (1) + g_3 (1) - b_{23} (2) - R_2 (g)) + p_2 u_3 (w_3 (2) + g_3 (2) - b_{23} (2)) \end{aligned}$$

Since $g + b_{23} \succ_2 g$, it must be that

$$\begin{aligned} & p_1 u_2 (w_2 (1) + g_2 (1)) + p_2 u_2 (w_2 (2) + g_2 (2)) \\ < \; & p_1 u_2 (w_2 (1) + g_2 (2) + b_{23} (2)) + p_2 u_2 (w_2 (2) + g_2 (2) + b_{23} (2)) \\ = \; & p_1 u_2 (w_2 (1) + g_2 (1) + R_2 (g) + b_{23} (2)) + p_2 u_2 (w_2 (2) + g_2 (2) + b_{23} (2)) \end{aligned}$$

In words, agent 3 pays (receives from) agent 2 an amount $b_{23} (2) + R_2 (g)$ ($b_{23} (2)$) in state 1 (2) in an economy where manipulation is impossible. Since the marginal rate of substitution between consumption in both states is identical between agents 2 and 3, this is a contradiction to the assumption that $g + b_{23} \succ_3 g$ and $g + b_{23} \succ_2 g$.